Impacts of Climate Change and Climate Policies

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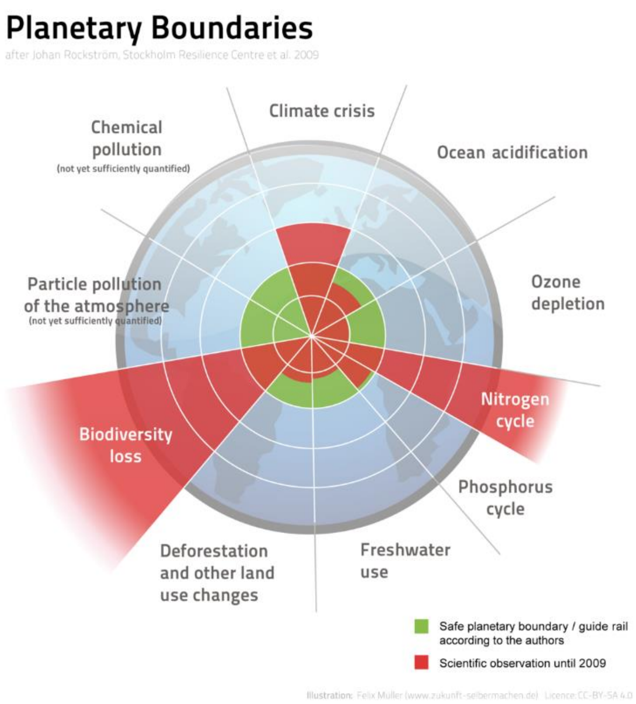
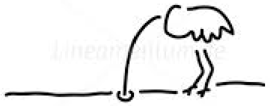
Environment and Climate Hub UNIVIE

The world is full of bad news about the environment and the economy



We can …

- Ignore or Deny



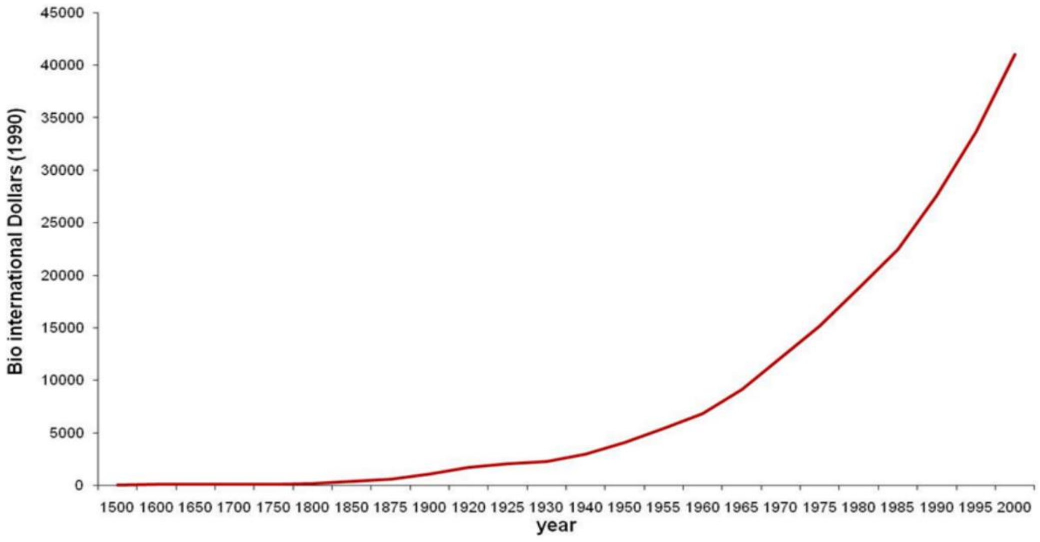
- Be pessimistic or optimistic

- Be alarmist

- Ask **scientifically** which real challenges we are facing and how to **provide solutions**

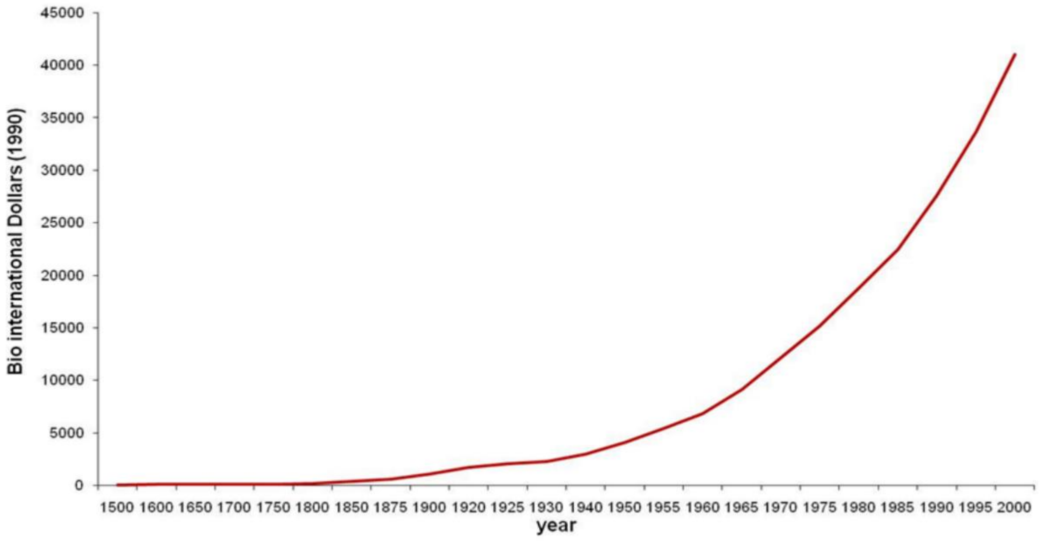
Basic Macro Facts

World Real GDP

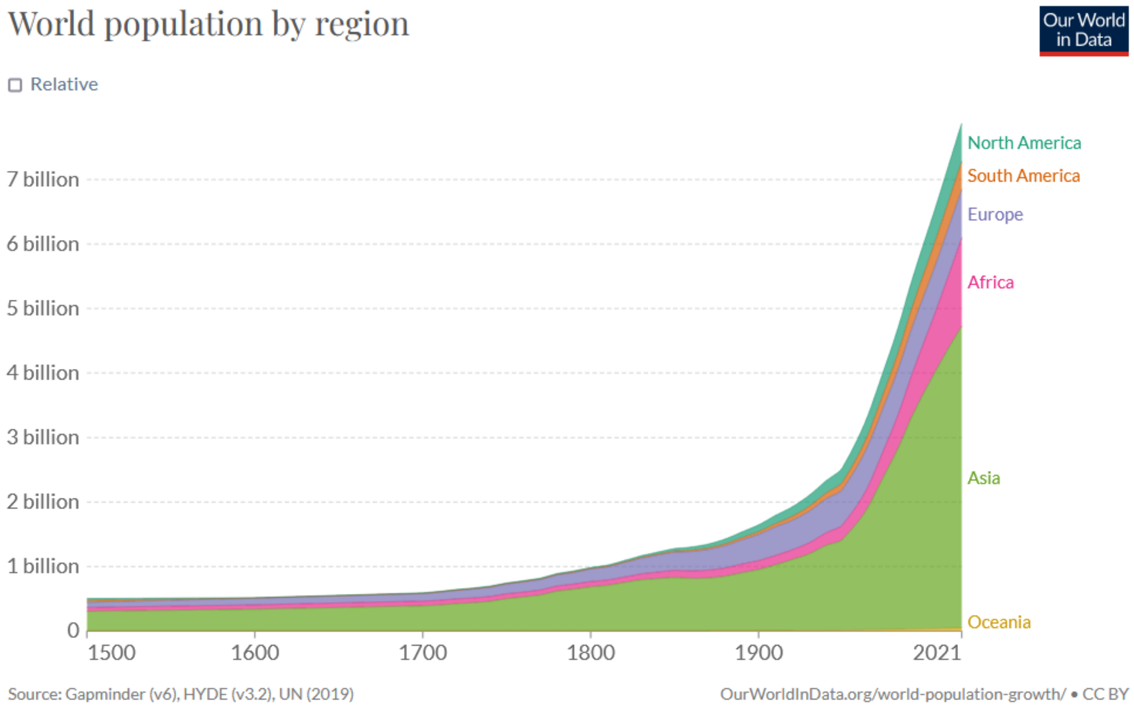


Basic Macro Facts

World Real GDP



World Population



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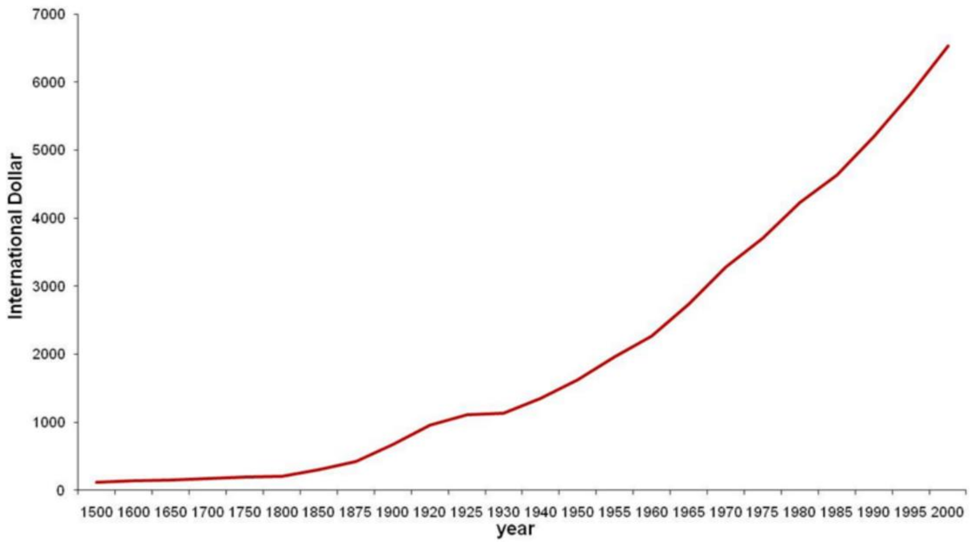
Basic Macro Facts

World Real GDP per capita



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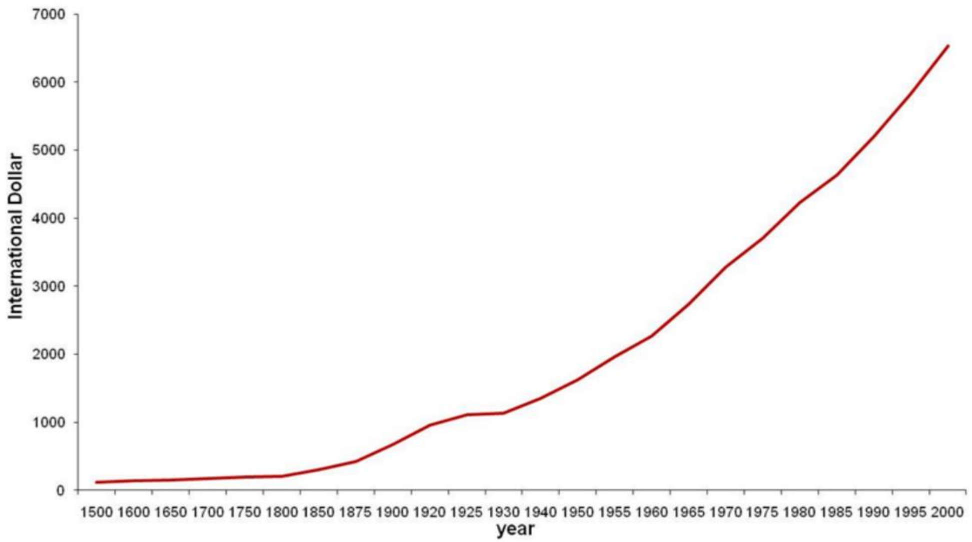
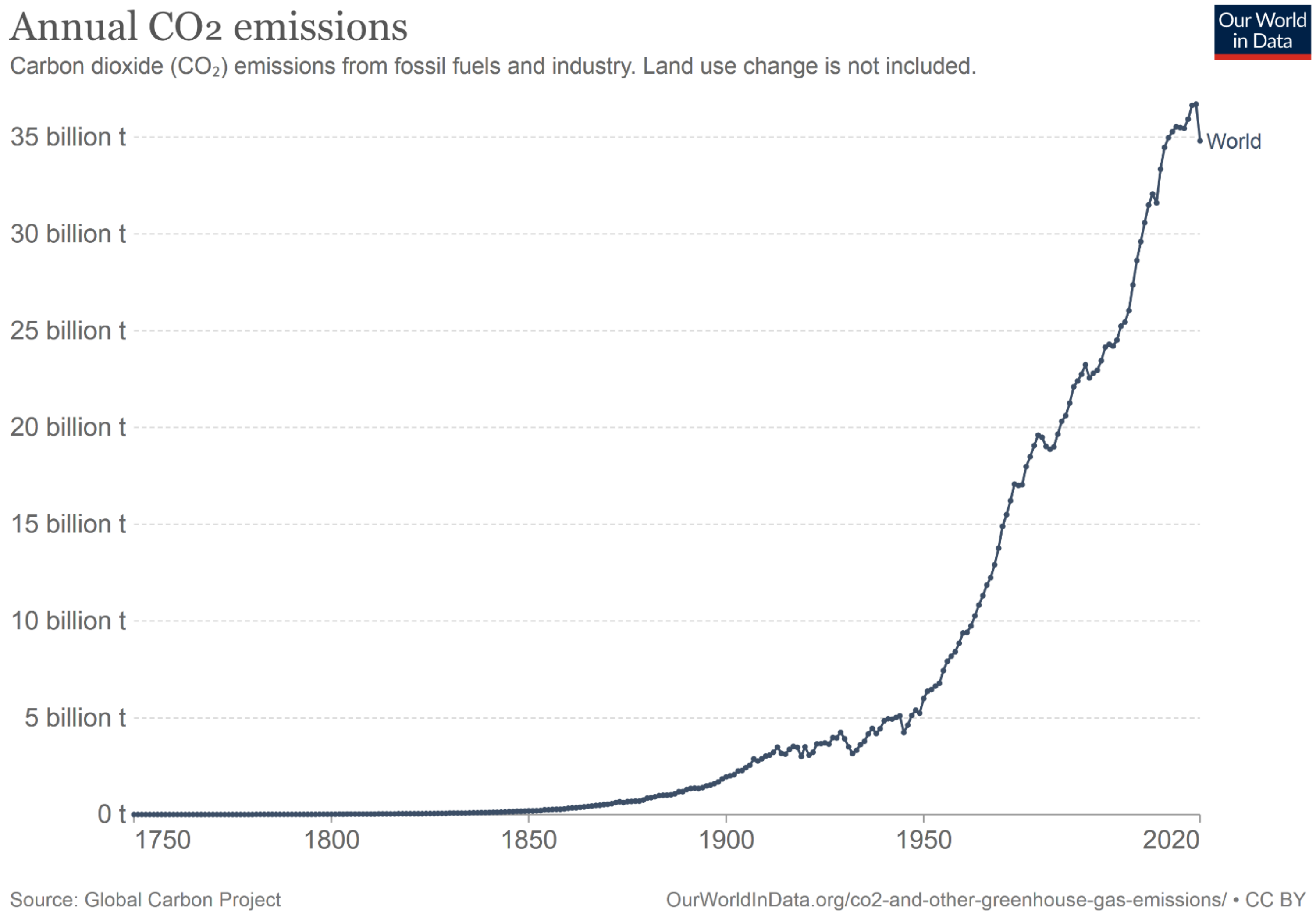
Basic Macro Facts

World Real GDP per capita



Finite Planet

CO_2 Emissions



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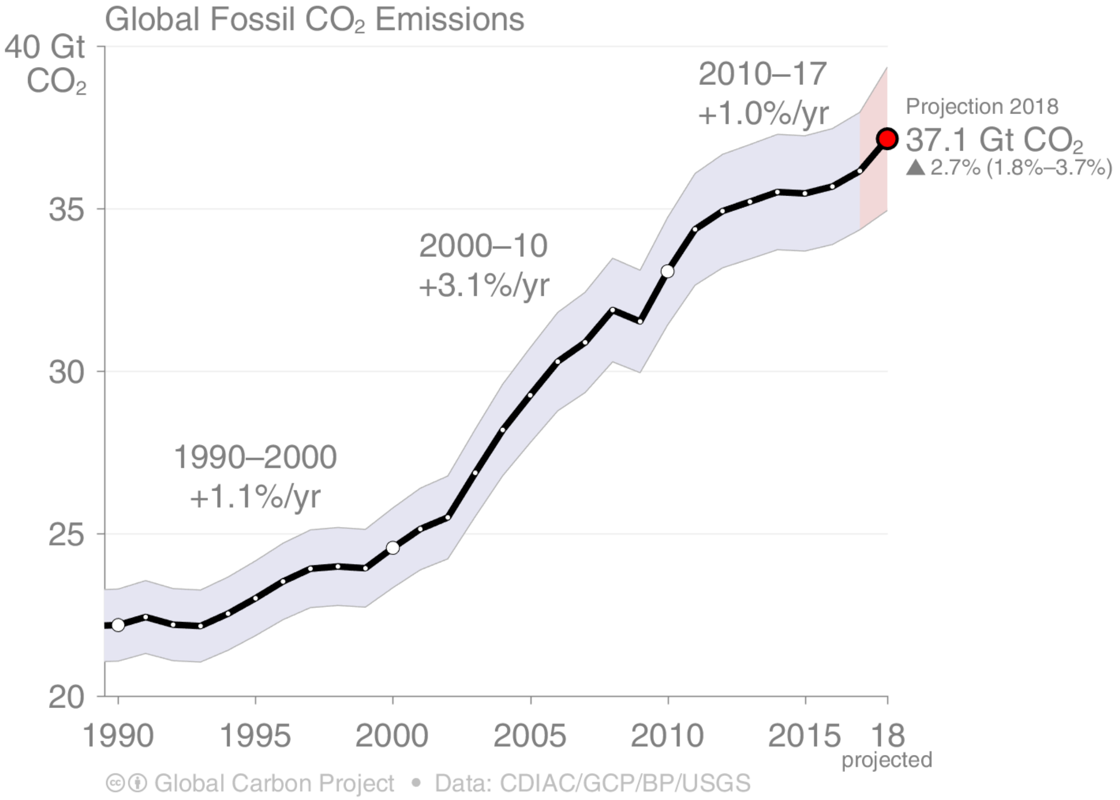
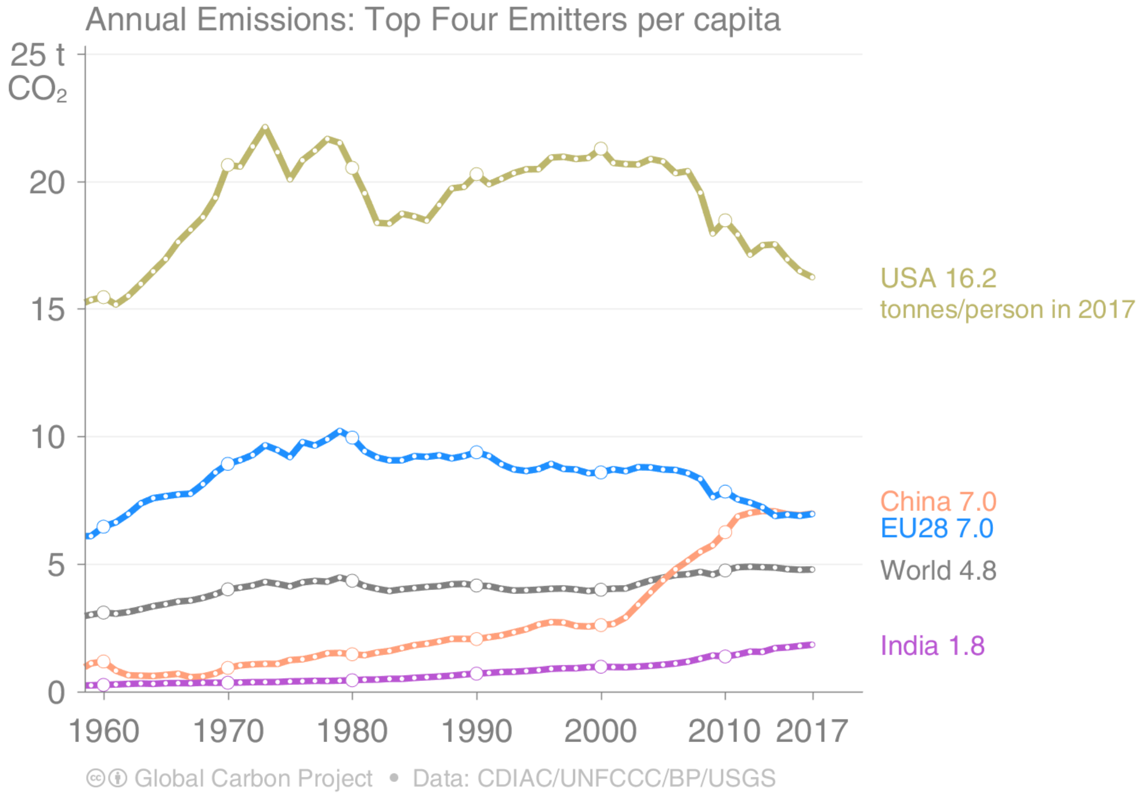
CO_2 Emissions





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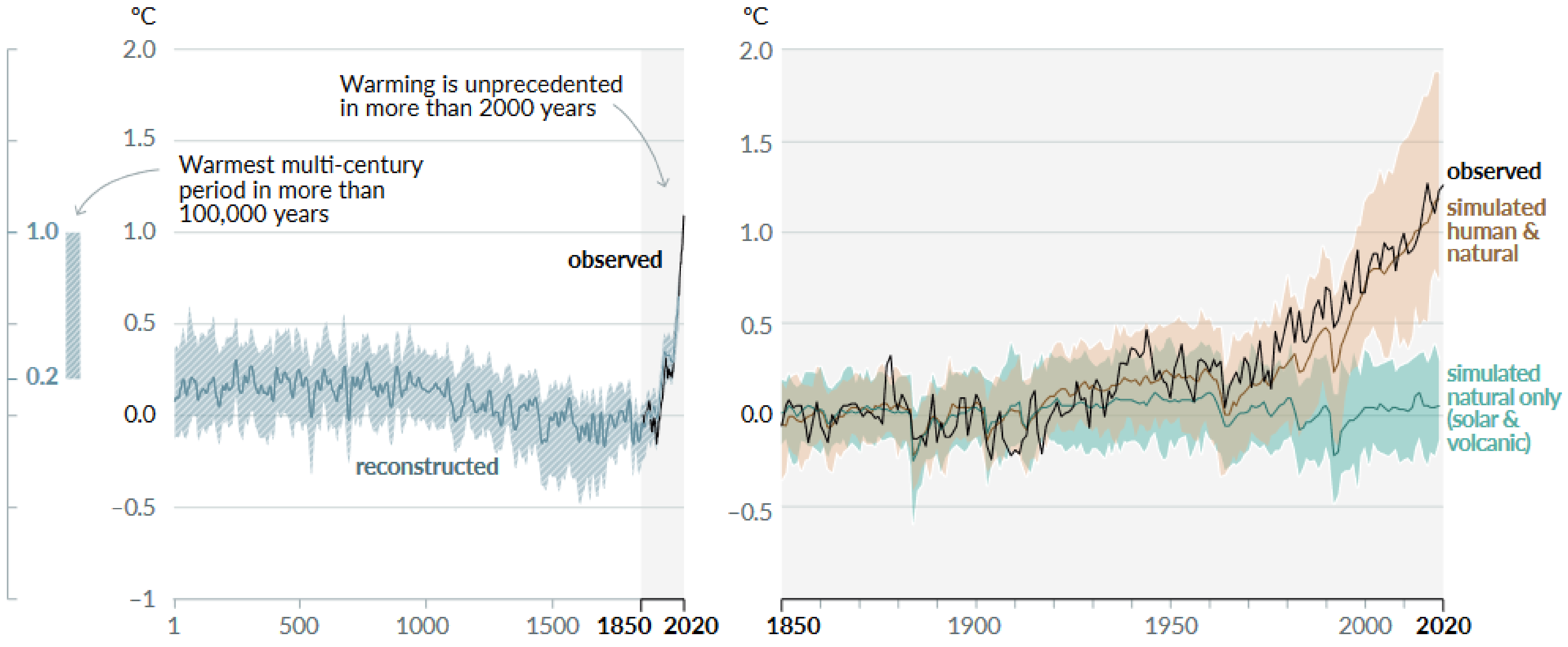
Source: [CDIAC]; [Le Quéré et al 2018]; [Global Carbon Budget 2018]

CO_2 Emissions and Warming



Source: IPCC 6th Assessment report

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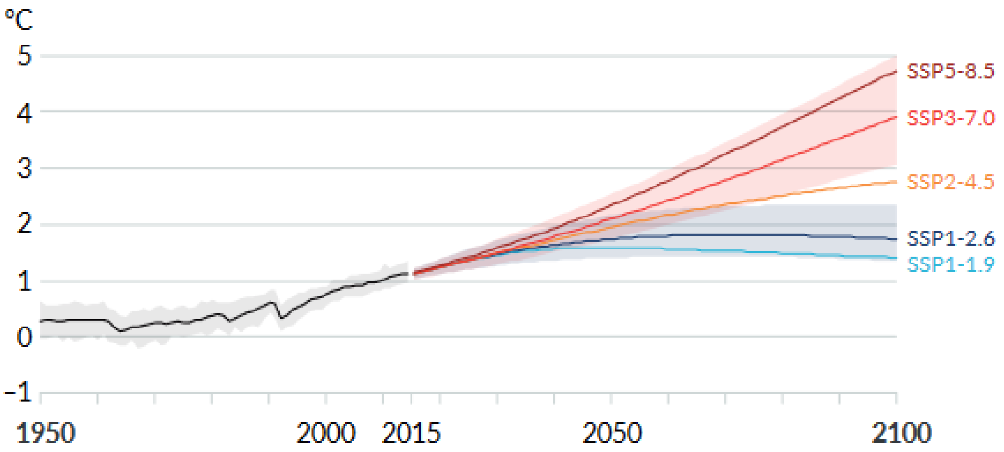
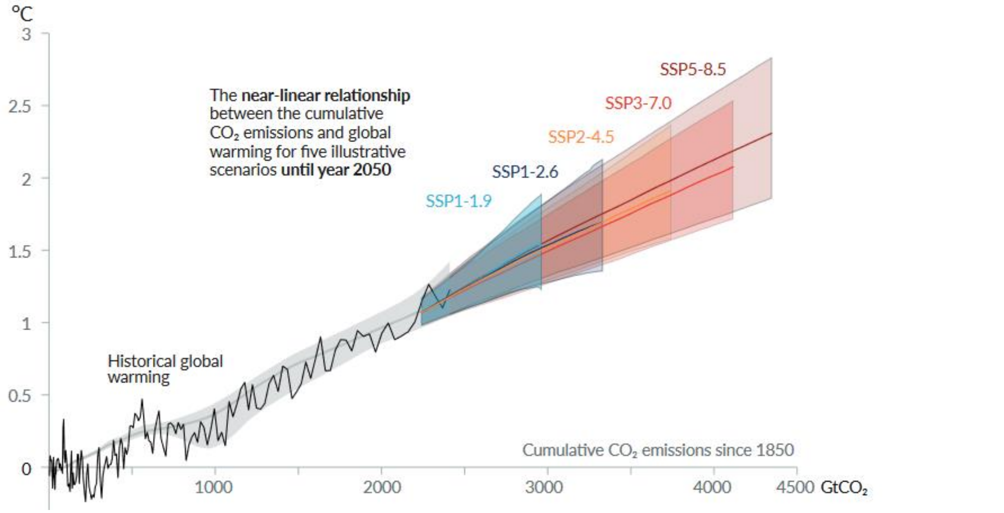
Climate Change

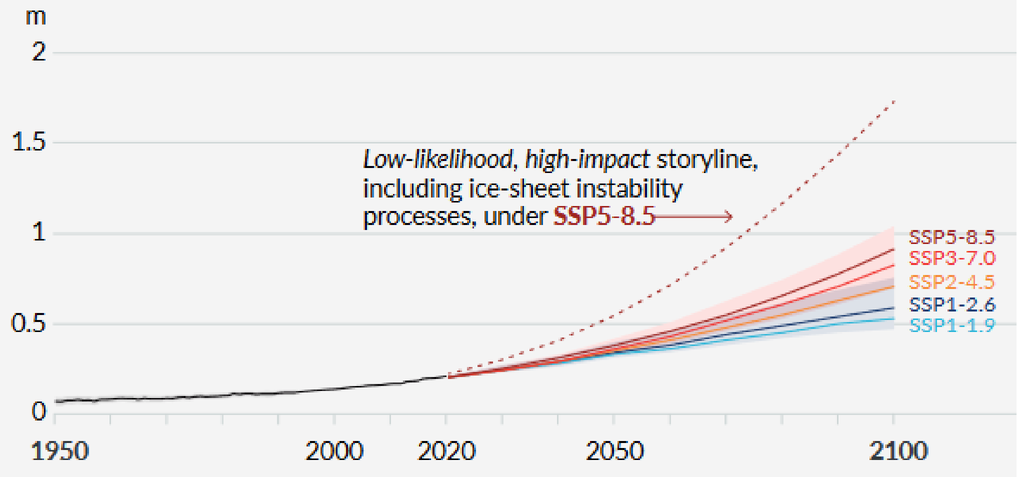
(a) Global surface temperature change relative to 1850–1900



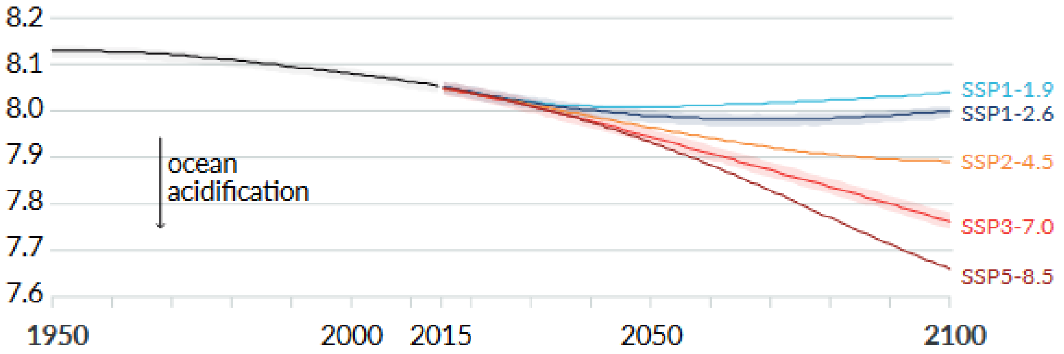
Global surface temperature increase since 1850–1900 (°C) as a function of cumulative CO_2 emissions ($GtCO_2$)



(d) Global mean sea level change relative to 1900



(c) Global ocean surface pH (a measure of acidity)



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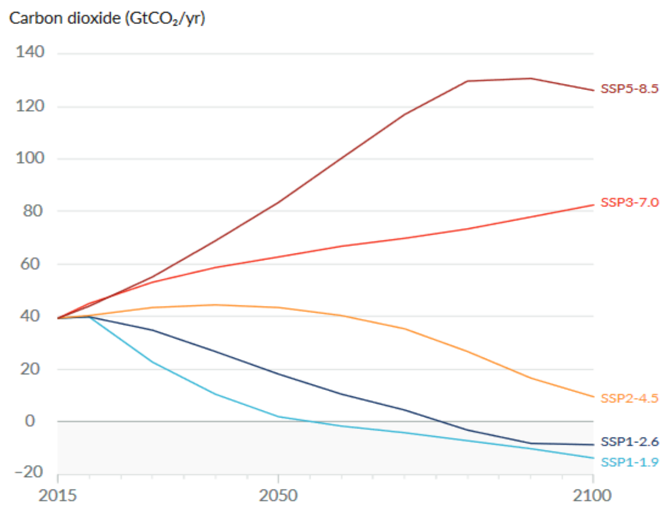
Climate Scenarios



	Near term, 2021–2040		Mid-term, 2041–2060		Long term, 2081–2100	
Scenario	Best estimate (°C)	*Very likely* range (°C)	Best estimate (°C)	*Very likely* range (°C)	Best estimate (°C)	*Very likely* range (°C)
SSP1-1.9	1.5	1.2 to 1.7	1.6	1.2 to 2.0	1.4	1.0 to 1.8
SSP1-2.6	1.5	1.2 to 1.8	1.7	1.3 to 2.2	1.8	1.3 to 2.4
SSP2-4.5	1.5	1.2 to 1.8	2.0	1.6 to 2.5	2.7	2.1 to 3.5
SSP3-7.0	1.5	1.2 to 1.8	2.1	1.7 to 2.6	3.6	2.8 to 4.6
SSP5-8.5	1.6	1.3 to 1.9	2.4	1.9 to 3.0	4.4	3.3 to 5.7

What Does It Really Mean?

Climate impacts on the economy and risks:

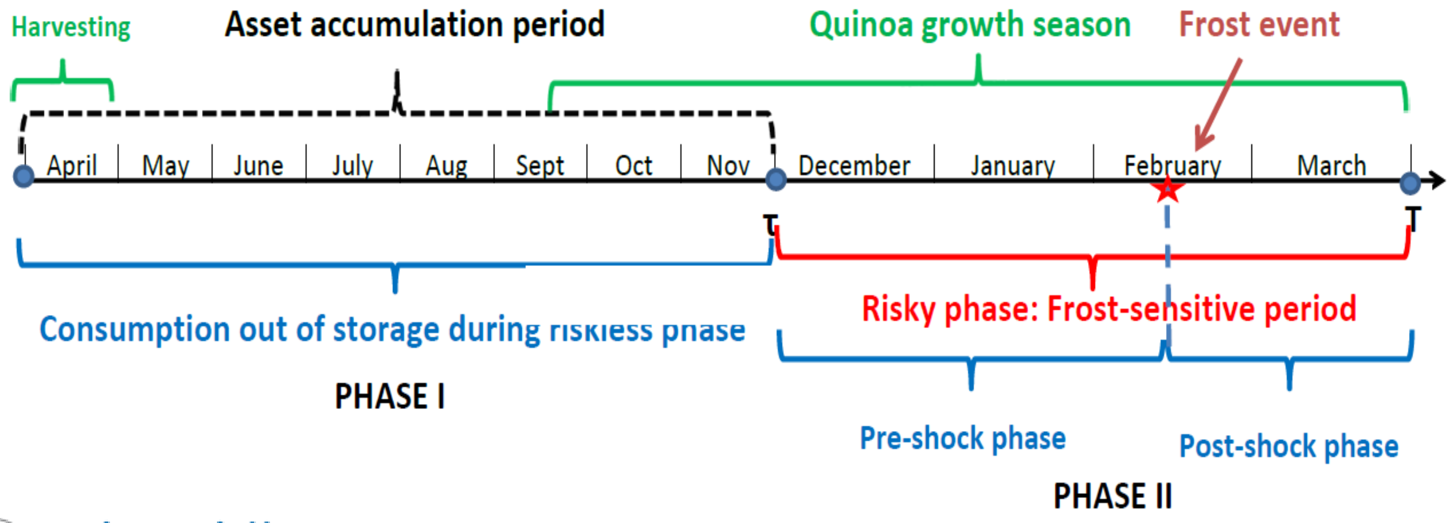
1. **Short** term (low damage): fluctuations in weather and hazards (frost, hail, heat wave, etc.)

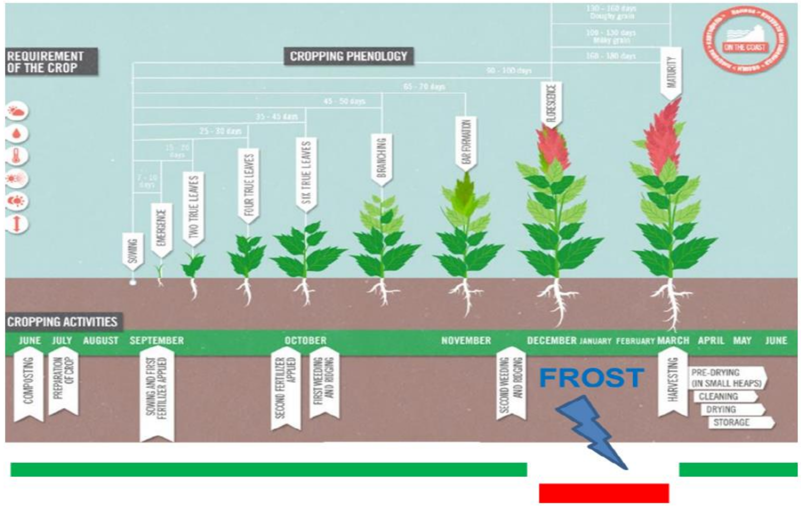
2. **Medium** term (substantial damage): changes in climate (ENSO, shifts in rainfall patterns, monsoon, natural disasters)

3. **Long** term (catastrophic): tipping points (currently 9 identified)

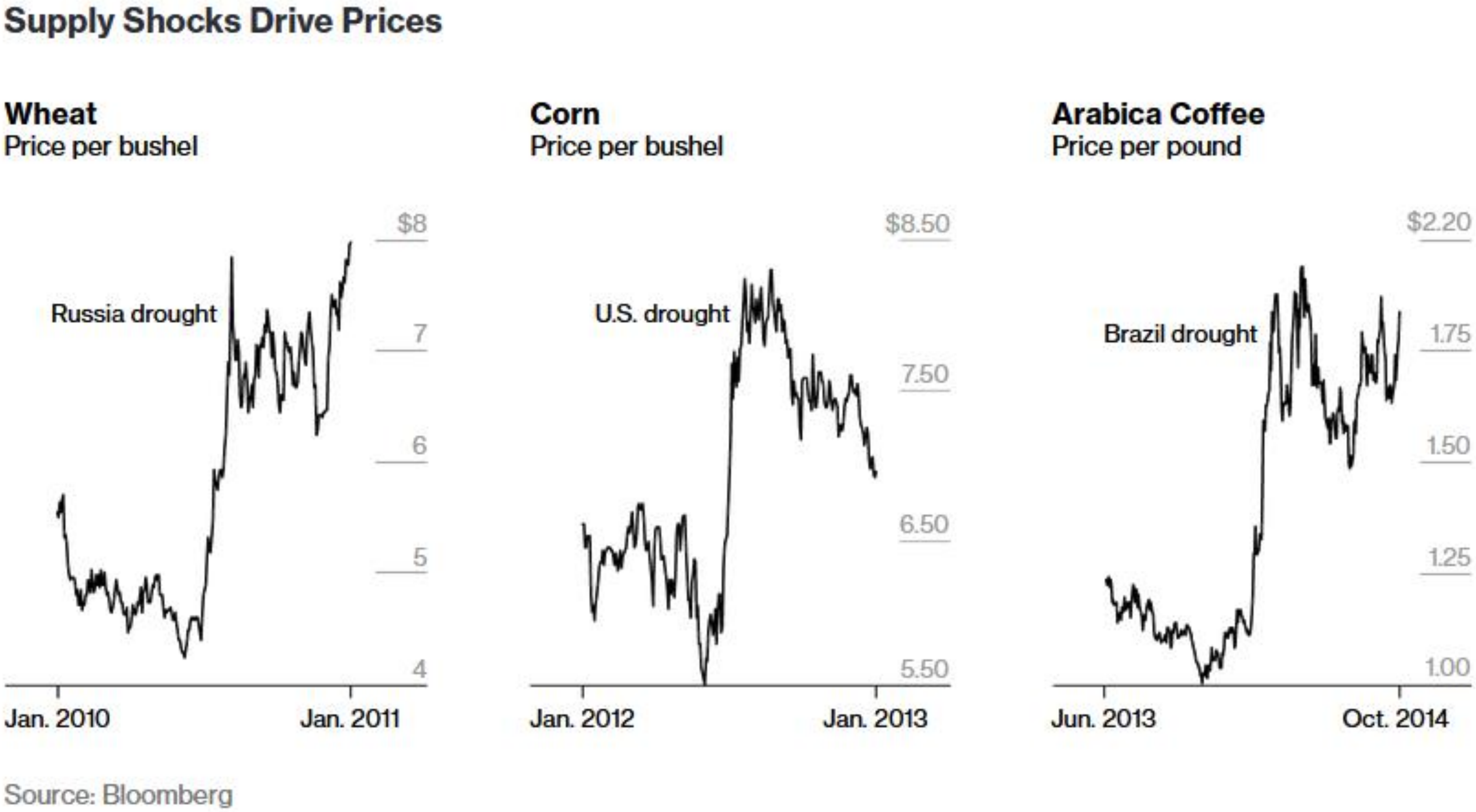
Short-term Impacts

- Mostly relevant for agriculture
- Dealt with by *adaptation* (climate services, IoT)





Short-term Impacts



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Adaptation - New Technologies

- Recent developments of new technologies can be extremely helpful for adaptation, e.g. IoT

- For example, new communication technologies, capable of functioning “in the wild” can be used for hyper-local forecasting

- Importantly, there is an unexploited potential of coupling several adaptation mechanisms together, e.g. insurance + CS + remote sensing

Medium-term Impacts

- Relevant for entire economy, potential contagion
- Dealt with by *mitigation* (CCS, carbon tax)

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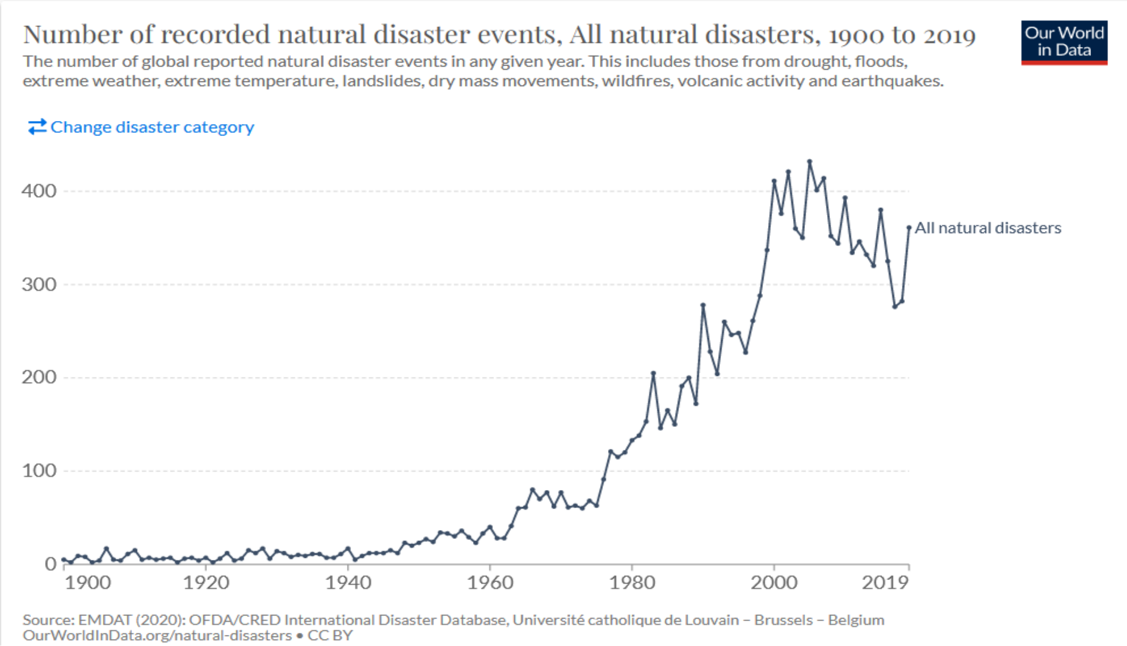
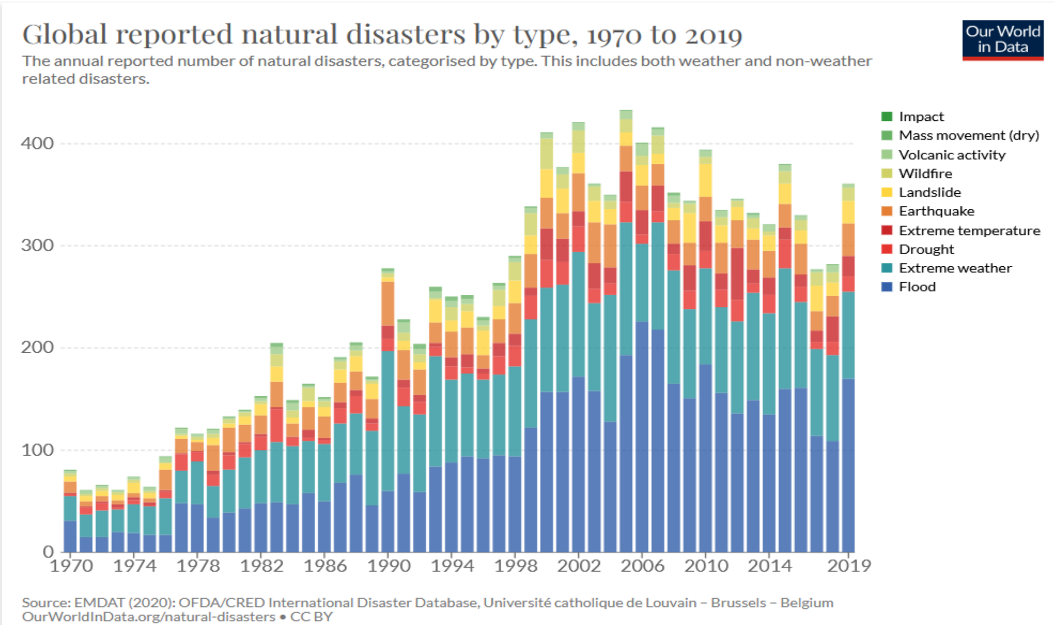
Climate-driven Disasters

- Frequency and intensity of disasters will rise (IPCC)





- Vulnerability of the economy (poor vs rich), widening inequality

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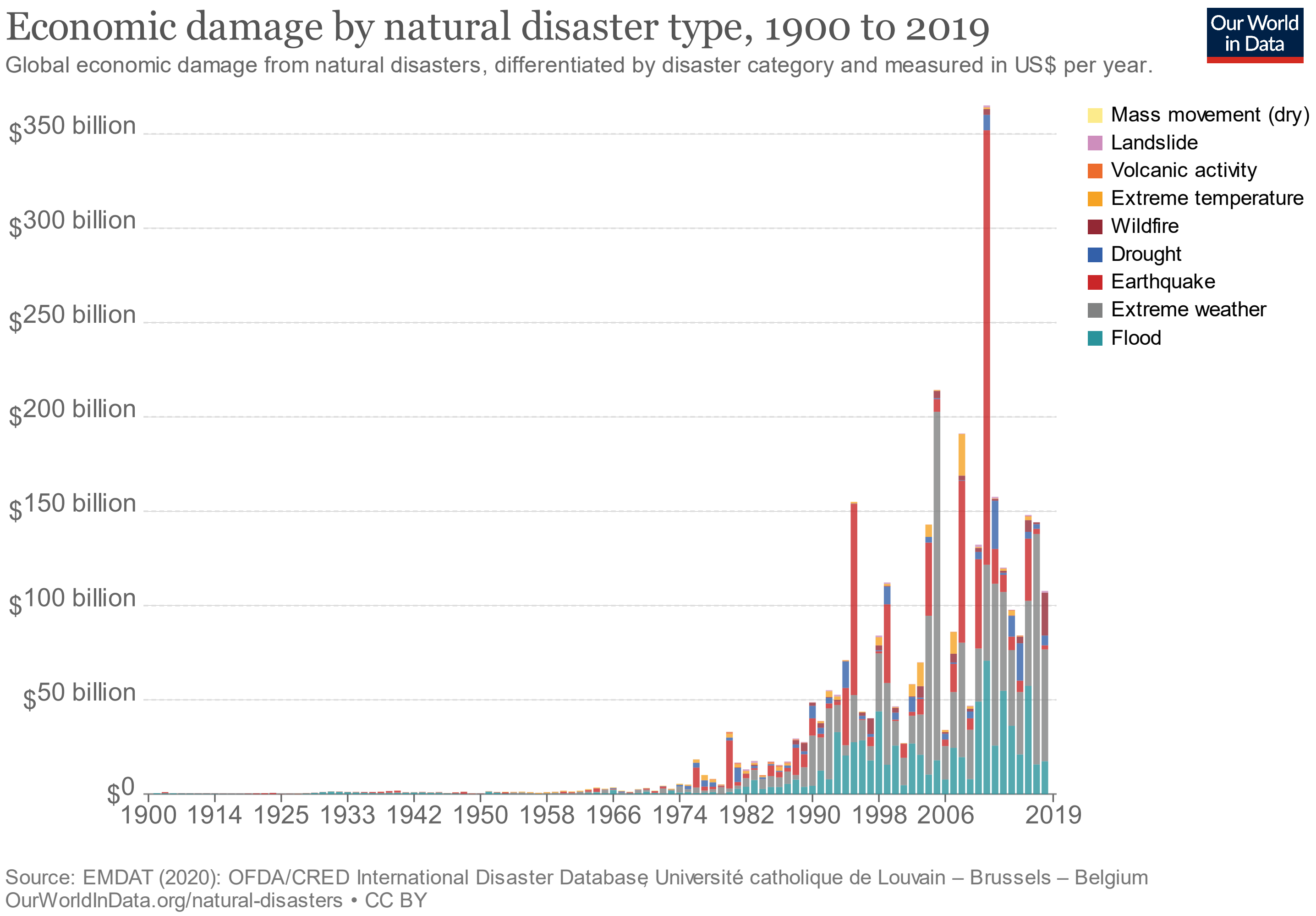
Losses from Disasters



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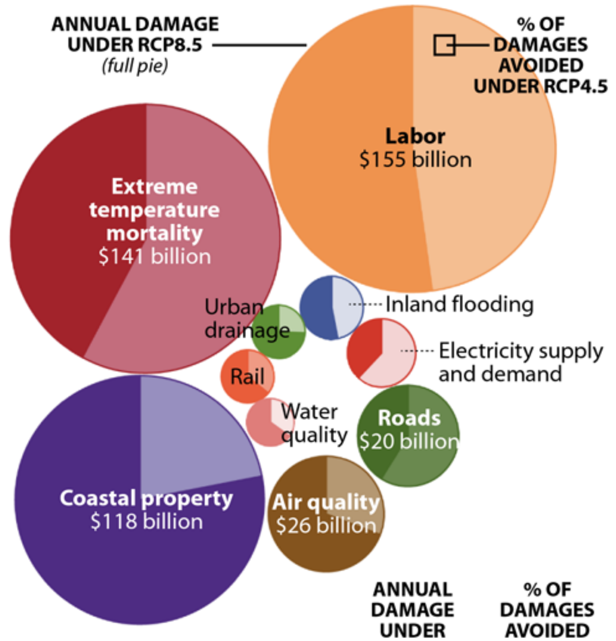
Overall Losses

FROM THE **2018 NATIONAL CLIMATE ASSESSMENT**

Climate Change's Economic Impact

The National Climate Assessment warns that the costs of global warming are rising. If greenhouse gas emissions continue at a high rate (RCP 8.5), damage from climate change is expected to cost the U.S. economy hundreds of billions of dollars every year by 2090. If emissions peak before mid-century and start down (RCP 4.5), the U.S. economy will still suffer, but the cost will be less. The chart shows some of the top economic expenses.



SECTOR	ANNUAL DAMAGE UNDER RCP8.5	% OF DAMAGES AVOIDED UNDER RCP4.5
Labor	$155B	48%
Extreme temperature mortality	$141B	58%
Coastal property	$118B	22%
Air quality	$26B	31%
Roads	$20B	59%
Electricity supply and demand	$9B	63%
Inland flooding	$8B	47%
Urban drainage	$6B	26%
Rail	$6B	36%
Water quality	$5B	35%

SOURCE: Fourth National Climate Assessment — InsideClimate News

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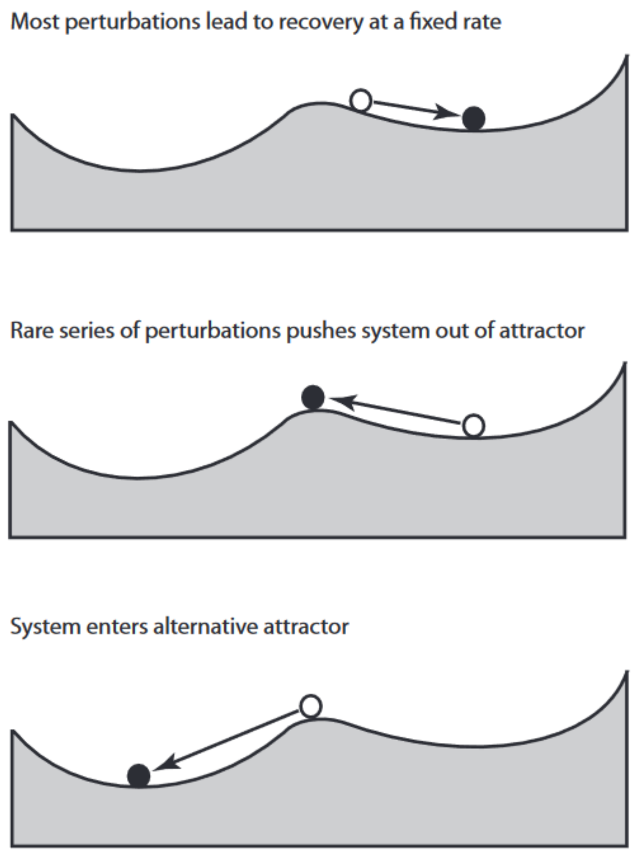
Tipping Points

- Tipping points produce abrupt system-wide change that is often difficult (and sometimes impossible) to reverse, giving them high impacts.
- Thus, even if their likelihood is low, they pose significant risks (risk is the product of the likelihood of an event and its impacts).
- Tipping points are difficult to predict, making them hard to manage.



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Lenton, T.M. (2013). „Environmental Tipping Points," *The Annual Review of Environment and Resources*, 38: 1 – 29.

Tipping Points

1. Atlantic thermohaline circulation (THC), reorganization
2. Greenland ice sheet (GIS), irreversible meltdown
3. West Antarctic ice sheet (WAIS), disintegration
4. Indian summer monsoon (ISM), disruption
5. West African monsoon (WAM), collapse
6. El-Nino Southern Oscillation (ENSO), increased amplitude
7. Arctic summer sea ice, abrupt loss
8. Amazon rainforest, dieback
9. Borel forest, dieback

Lenton, T.M. (2013). „Environmental Tipping Points," *The Annual Review of Environment and Resources*, 38: 1 – 29.
Lenton, T.M., and J.-C. Ciscar (2013). „Integrating Tipping Points into Climate Impact Assessment," *Climatic Change*, 117: 585 - 597

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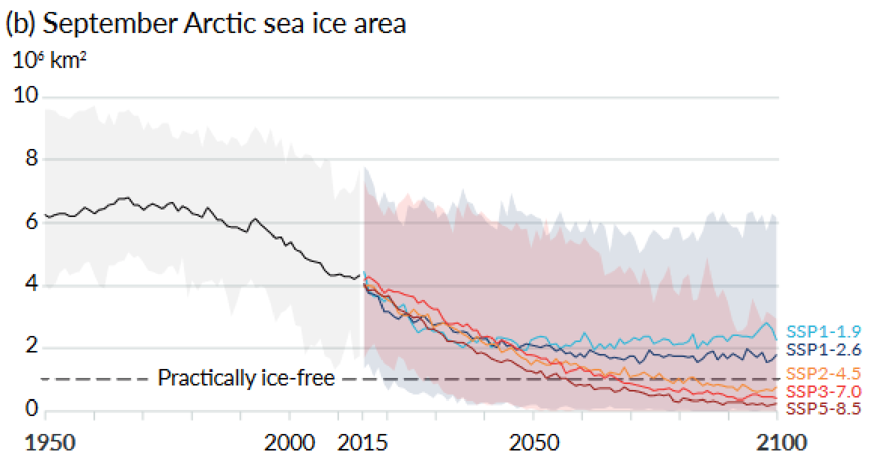
Tipping Points: Uncertainties



- Pollution threshold is uncertain
- Tipping may occur even at low warming
- Nearest candidates: Arctic ice sheet & GIS: 0.5 – 2° C
- Most of the others: 3 – 5°
- >16% prob of 1 tipping under medium warming (2 – 4°), >56% prob of tipping all under high warming (>4°)

Lenton, T.M. (2013). „Environmental Tipping Points," *The Annual Review of Environment and Resources*, 38: 1 – 29.
Lenton, T.M., and J.-C. Ciscar (2013). „Integrating Tipping Points into Climate Impact Assessment," *Climatic Change*, 117: 585 - 597

Tipping Points: Impacts

- These „events“ are slow-onset, irreversible and high-damage
- Huge sea-level rise: Greenland ice sheet ~ 7m; West Antarctica ~ 3m
- Catastrophic (?): collapse of Atlantic THC
- Droughts: ENSO, ISM, WAM, Amazon, Borel
- Biodiversity loss: Amazon, Borel
- No concensus on monetary value: ~ at least 25% of GWP

Lenton, T.M., and J.-C. Ciscar (2013). „Integrating Tipping Points into Climate Impact Assessment,“ *Climatic Change*, 117: 585 - 597

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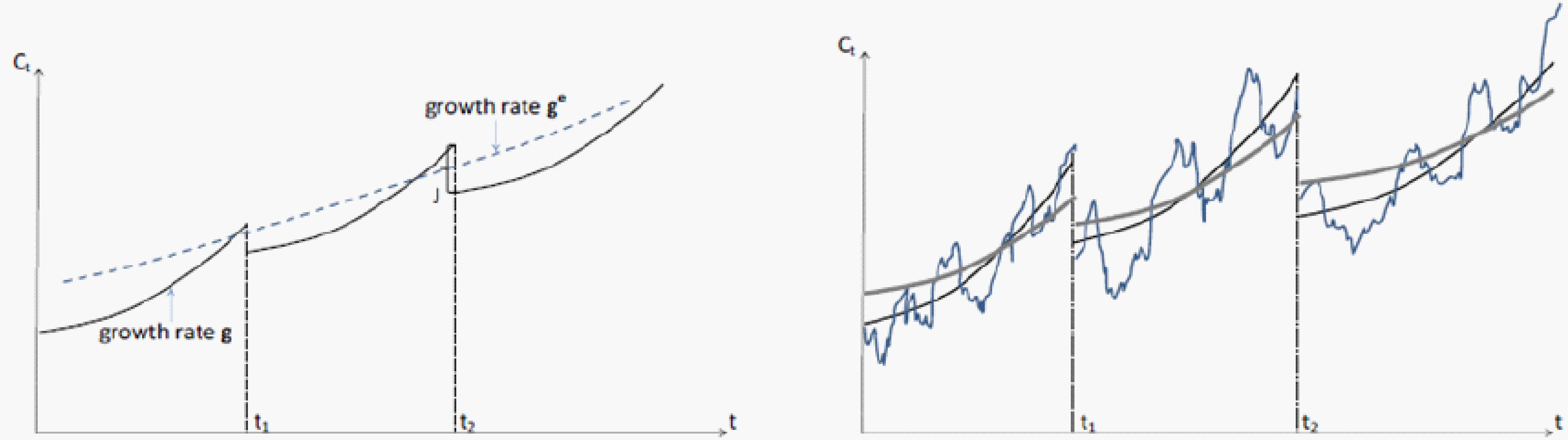
Climate Damages

- Damages to **productivity** or to **capital**? → Both!

- Capital destruction has a level and a growth effect



(a) Expected (dashed) vs. Poisson-driven (solid) growth.

(b) Poisson and Wiener uncertainty (thin gray) vs. stock effects (bold).

Bretschger and Vinogradova (2018)

Broadening the Scope

- Environmental migration
  - UN: in 2008, 20 mln people were displaced by climate change
  - Projected 250 mln by 2050
  - Migration to cities will increase, especially in the global South; vulnerability to sea-level rise
- Social impact (conflict, violence)
- Food systems
- Health (air, soils, water)

Environmental Migration

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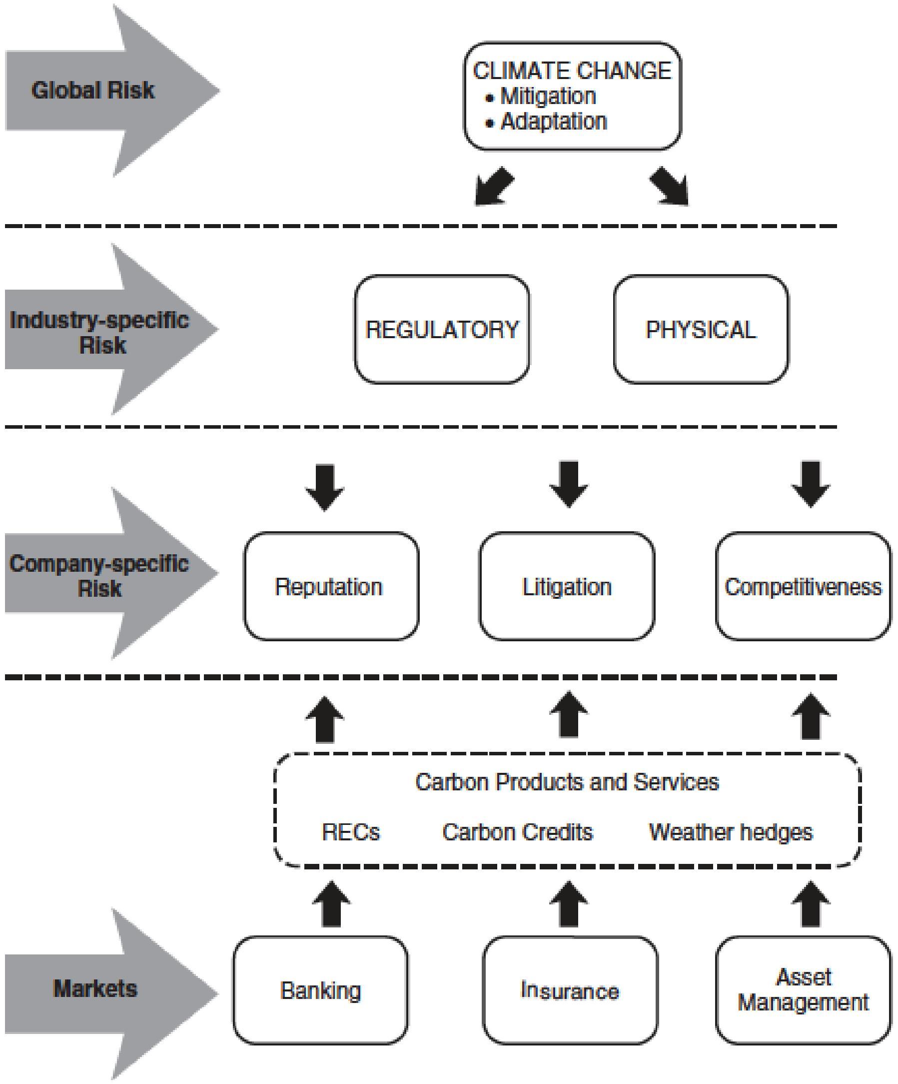
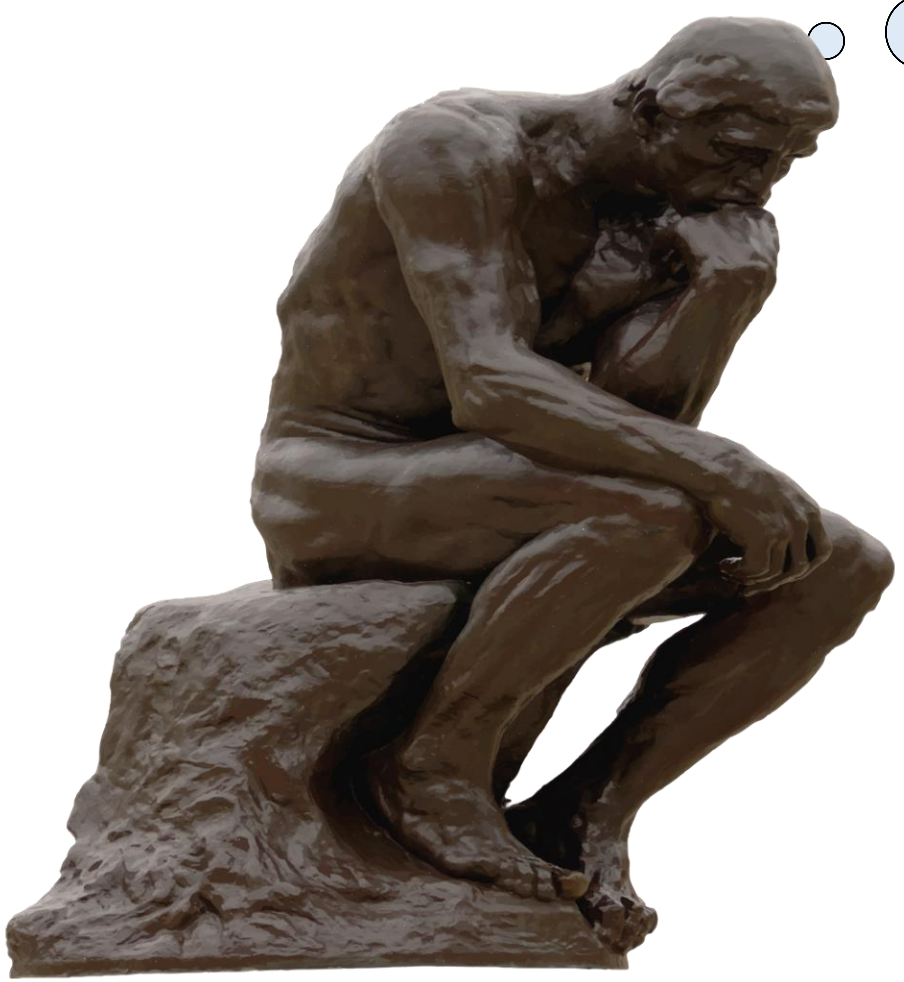
Big Picture



Source: Labatt&White, Carbon Finance

What to Do?

- What are the optimal policies to mitigate climate change and at the same time to ensure sustainable development?



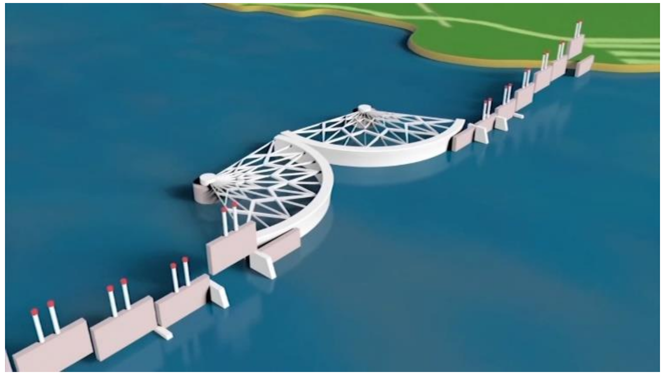
«Doubt is an uncomfortable condition
but certainty is a ridiculous one»

Voltaire

Climate Change

Mitigation

Adaptation



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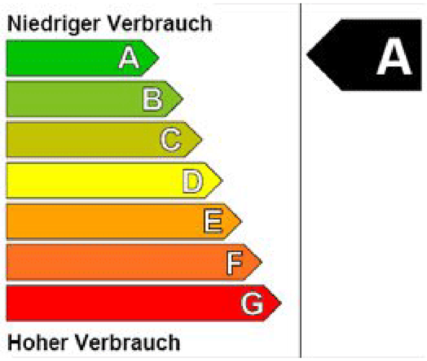
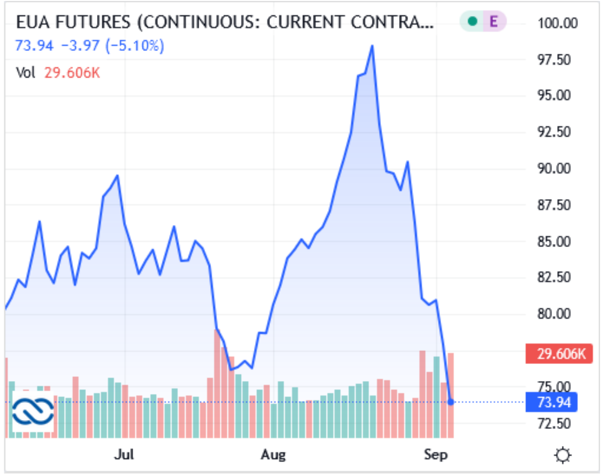
Available Policies



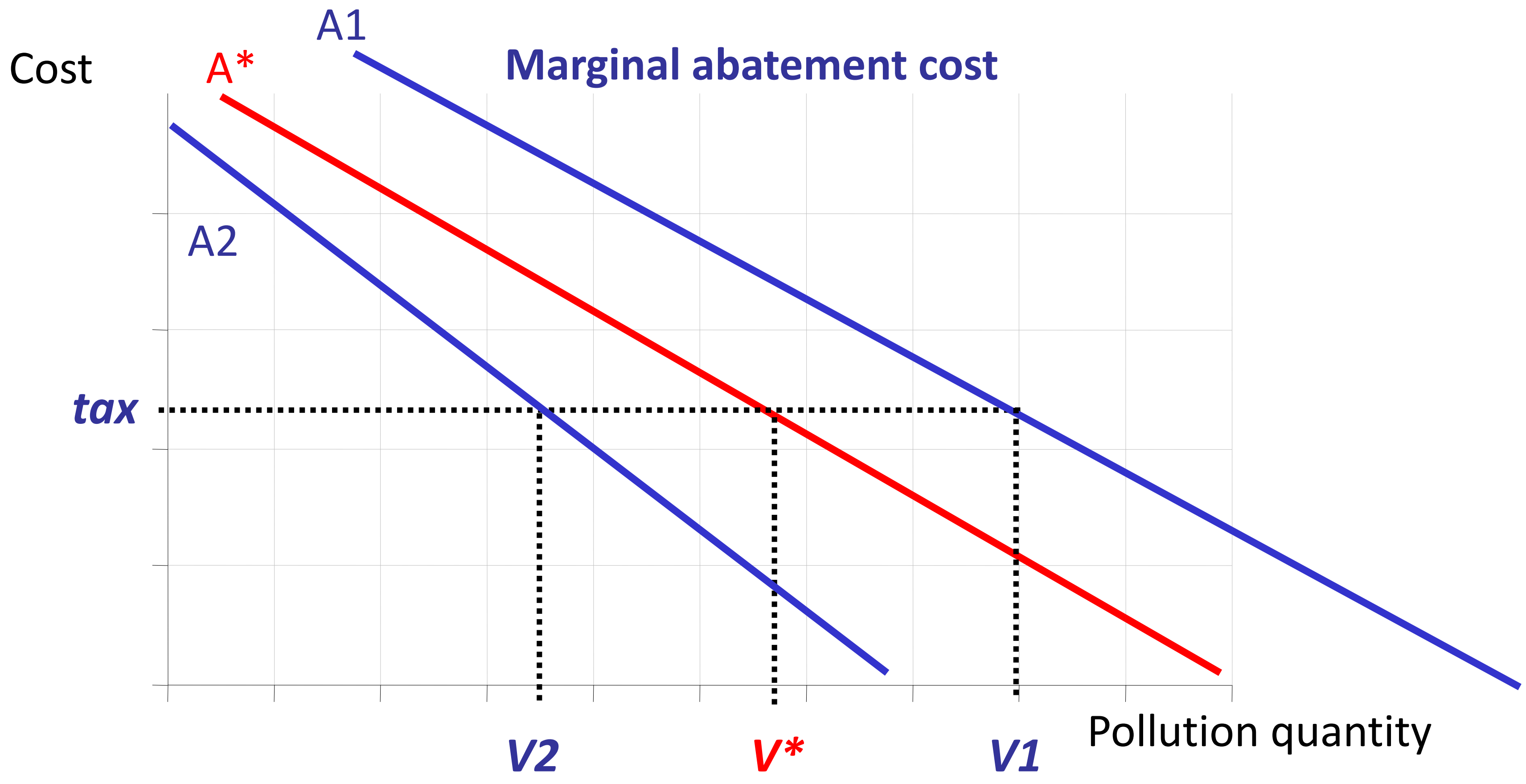
- Direct production of environmental quality
- Command and control instruments
  - Quotas
  - Standards
  - Technology controls
  - Inputs restrictions



- Market-based instruments
  - Carbon tax
  - Tradable permits

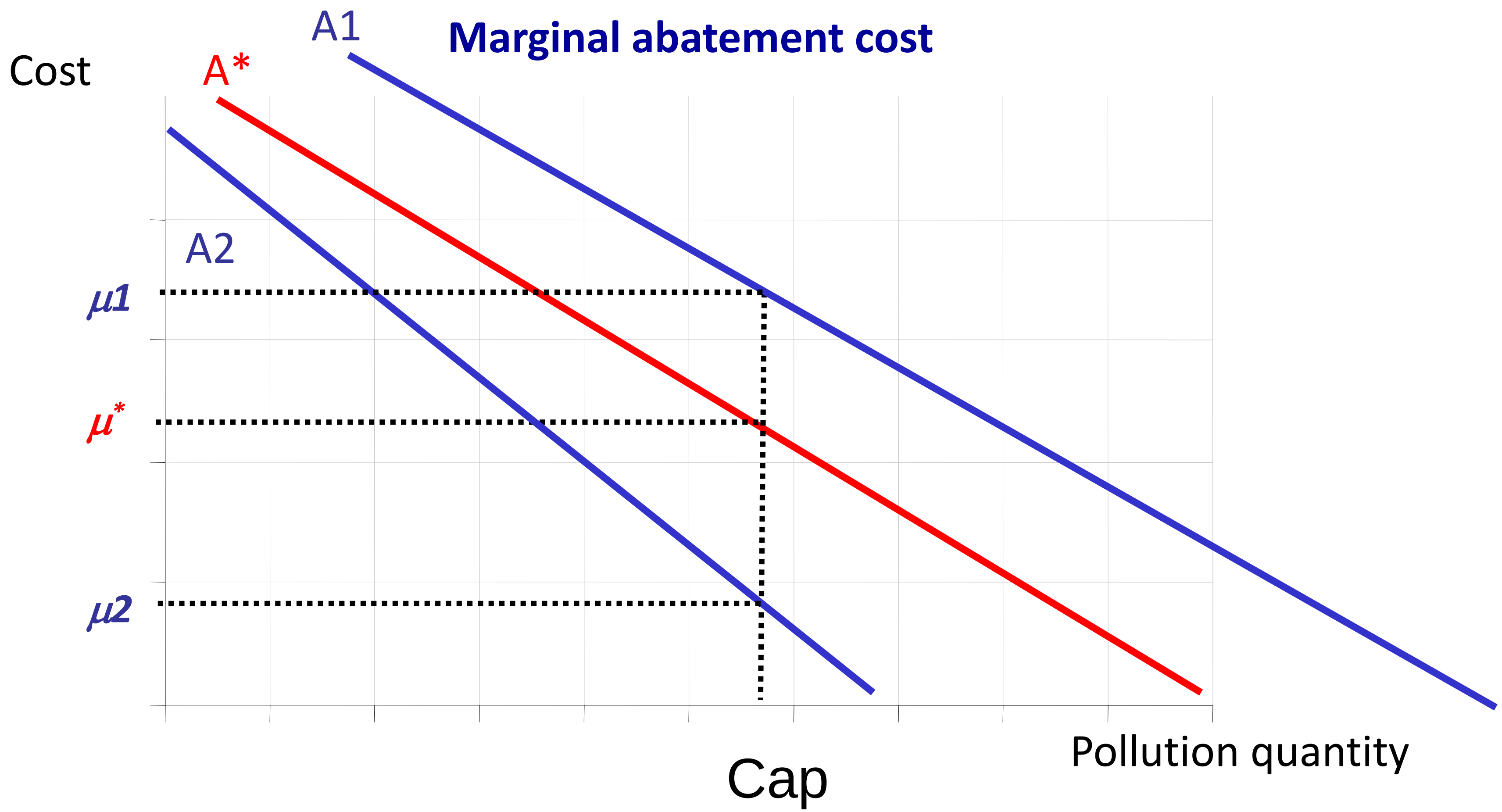
Carbon Tax vs ETS

Imperfect Information about abatement cost curve: Tax



Carbon Tax vs ETS

Imperfect Information about abatement cost curve: Permits



Quantitative Assessment

- Optimal abatement propensity (% GDP) / Carbon price
  - Low-impact events: low damage intensity of disasters (<1% consumption loss)
  - High-impact events: high damage intensity of disasters (up to 10% consumption loss)
  - Tipping points: severe (30% GDP loss), destructive (90% GDP loss)
- Variation in
  - Risk aversion
  - Abatement efficiency (\$12.5/tCO_2; \$20/tCO_2)
  - Event arrival rate (20% prob in next 10, 20, 50 years)

Bretschger and Vinogradova (2022)

Quantitative Assessment
Low-impact events

	$\varepsilon = 1$		$\varepsilon = 3$	
	σ =0.08	σ =0.05	σ =0.08	σ =0.05
λ =0.02				
θ	0.50075	0.80121	0.70644	1.13617
g	3.47496	3.45994	1.15486	1.14764
λ =0.01				
θ	0.50038	0.80060	0.60294	0.96733
g	3.47498	3.45997	1.15660	1.15059
λ =0.004				
θ	0.50015	0.80024	0.54111	0.86675
g	3.47499	3.45999	1.15764	1.15220

ε risk aversion

σ abatement efficiency

λ arrival probability

θ abatement, % GDP

g growth rate

Implied optimal carbon price \$37 - \$63/ tCO2

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Quantitative Assessment
High-impact events

	$\varepsilon = 1$		$\varepsilon = 3$	
	σ =0.08	σ =0.05	σ =0.08	σ =0.05
λ =0.02				
θ	0.50753	0.81205	2.90496	5.17775
g	3.47462	3.4540	1.11408	1.06784
λ =0.01				
θ	0.50377	0.80603	1.66156	2.84628
g	3.47481	3.45970	1.13702	1.11368
λ =0.004				
θ	0.50151	0.80241	0.95565	1.58971
g	3.47492	3.45988	1.14998	1.13809

ε risk aversion

σ abatement efficiency

λ arrival probability

θ abatement, % GDP

g growth rate

Implied optimal carbon price up to $392/ tCO2

Quantitative Assessment

Tipping points & low-impact events

	λ_2=0.00084		λ_2=0.00294	
	σ =0.08	σ =0.05	σ =0.08	σ =0.05
λ_1=0.02				
θ	0.74230	1.19549	0.87813	1.42279
g	1.65435	1.64672	2.90229	2.89302
λ_1=0.01				
θ	0.62074	0.99665	0.68791	1.10821
g	1.65640	1.65010	2.90552	2.89842
λ_1=0.004				
θ	0.54812	0.87840	0.57489	0.92254
g	1.65763	1.65211	2.90744	2.90161

ε	risk aversion
σ	abatement efficiency
λ	arrival probability
θ	abatement, % GDP
g	growth rate

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Quantitative Assessment

Tipping points & High-impact events

	λ_2=0.00084		λ_2=0.00294	
	σ =0.08	σ =0.05	σ =0.08	σ =0.05
λ_1=0.02				
θ	3.41946	6.30301	5.81557	13.75750
g	1.10369	1.04279	1.05203	0.85749
λ_1=0.01				
θ	1.89521	3.30787	2.89469	5.53646
g	1.13233	1.10354	1.11106	1.05049
λ_1=0.004				
θ	1.04431	1.75690	1.40935	2.49407
g	1.14821	1.13445	1.14049	1.11717

ε	risk aversion
σ	abatement efficiency
λ	arrival probability
θ	abatement, % GDP
g	growth rate

Implied optimal carbon price up to ~$1000/ tCO2

Multiple Disasters

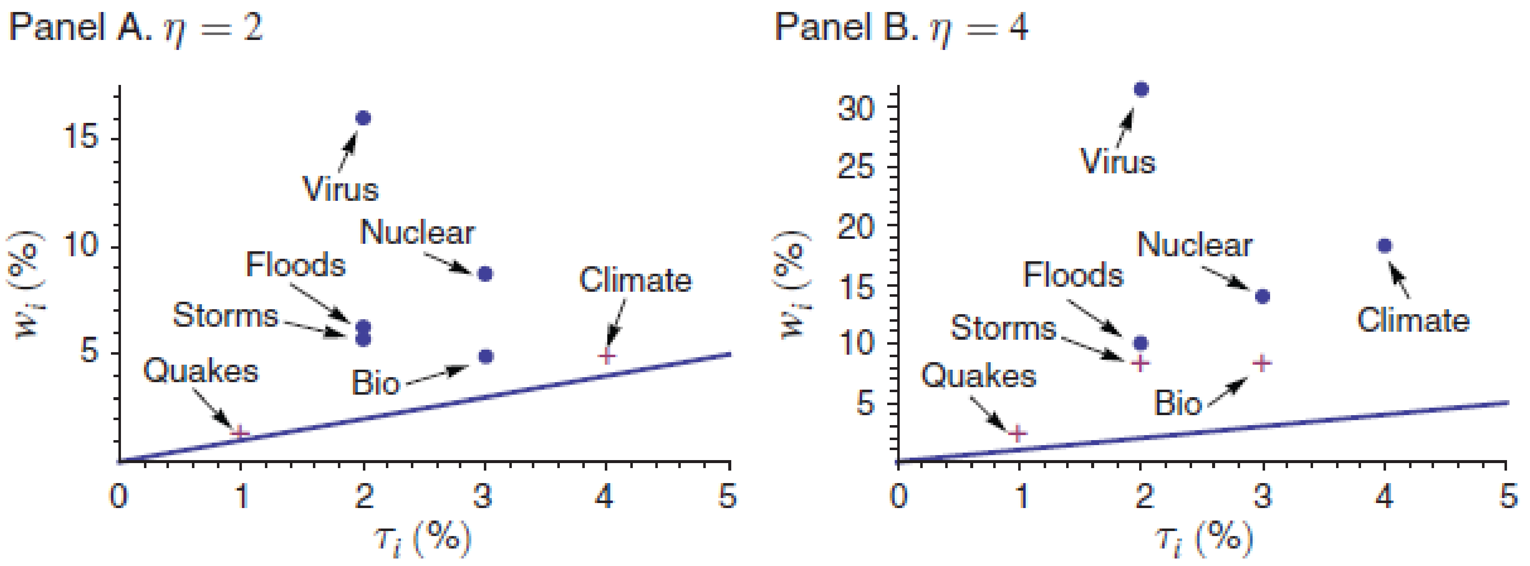


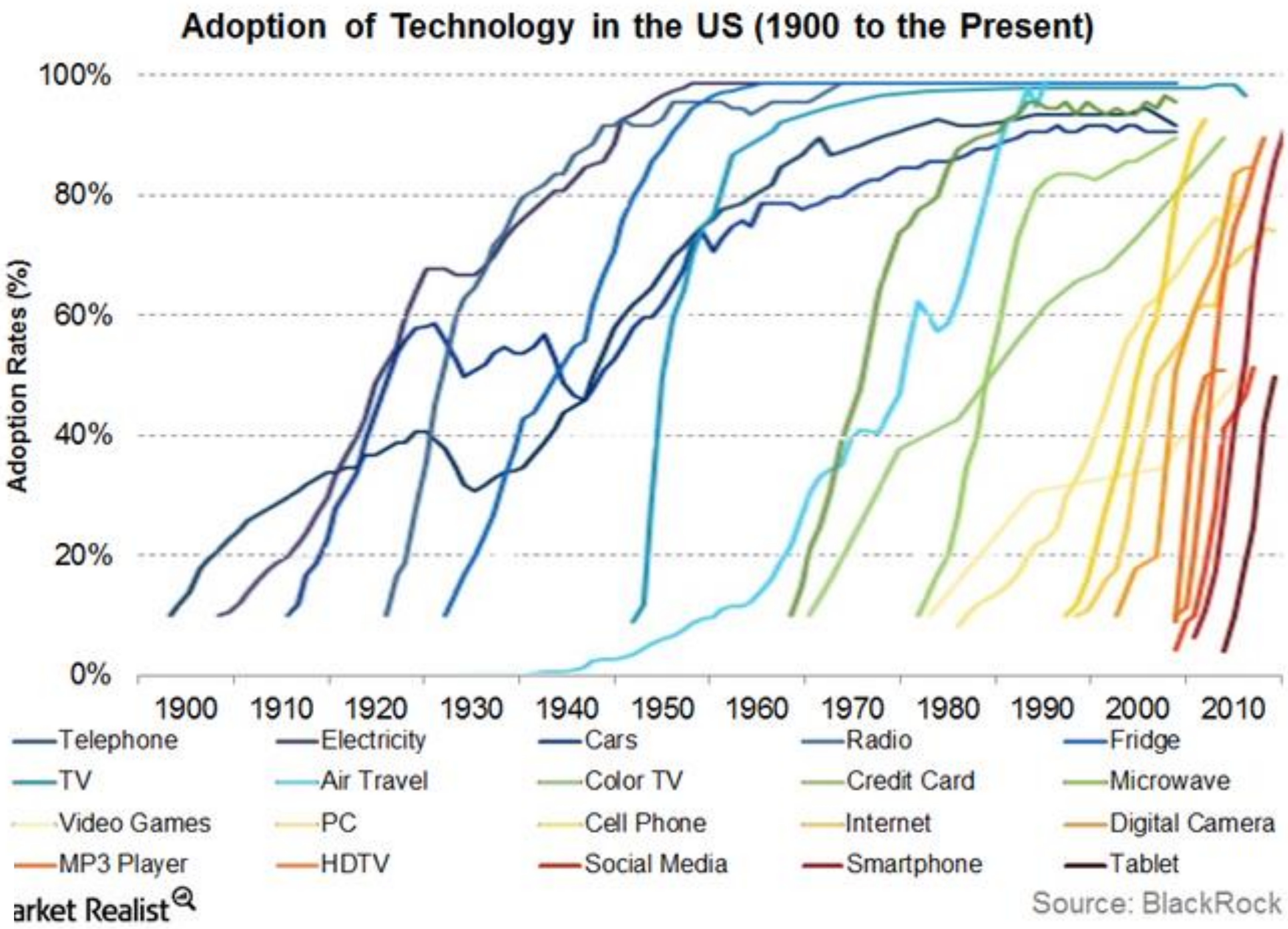
Figure 6

Notes: The figures show which of the seven catastrophes summarized in Table 1 should be averted. Catastrophes that should be averted are indicated by dots in each panel; catastrophes that should *not* be averted are indicated by crosses.

Why Carbon Tax May NOT Do the Job?

- Carbon budget: < 275 GtC to reach Paris target
- Fossil fuel reserves (oil, gas, coal): 440 GtC!
- Risk of stranded assets
- Other solutions?

Technology Adoption



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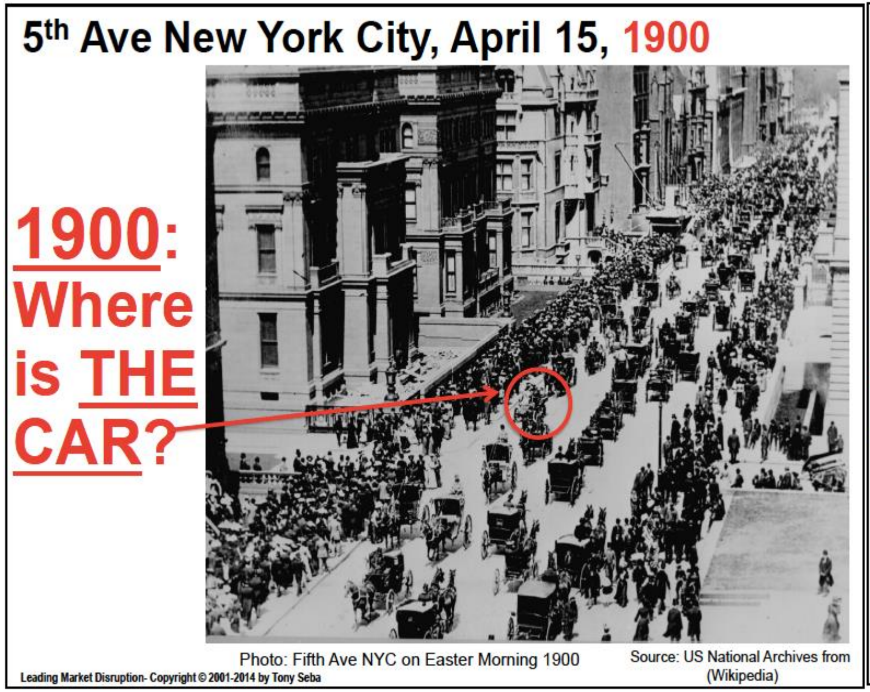
Disruptive Development



The Economist

2018

Electric cars

The death of the internal combustion engine

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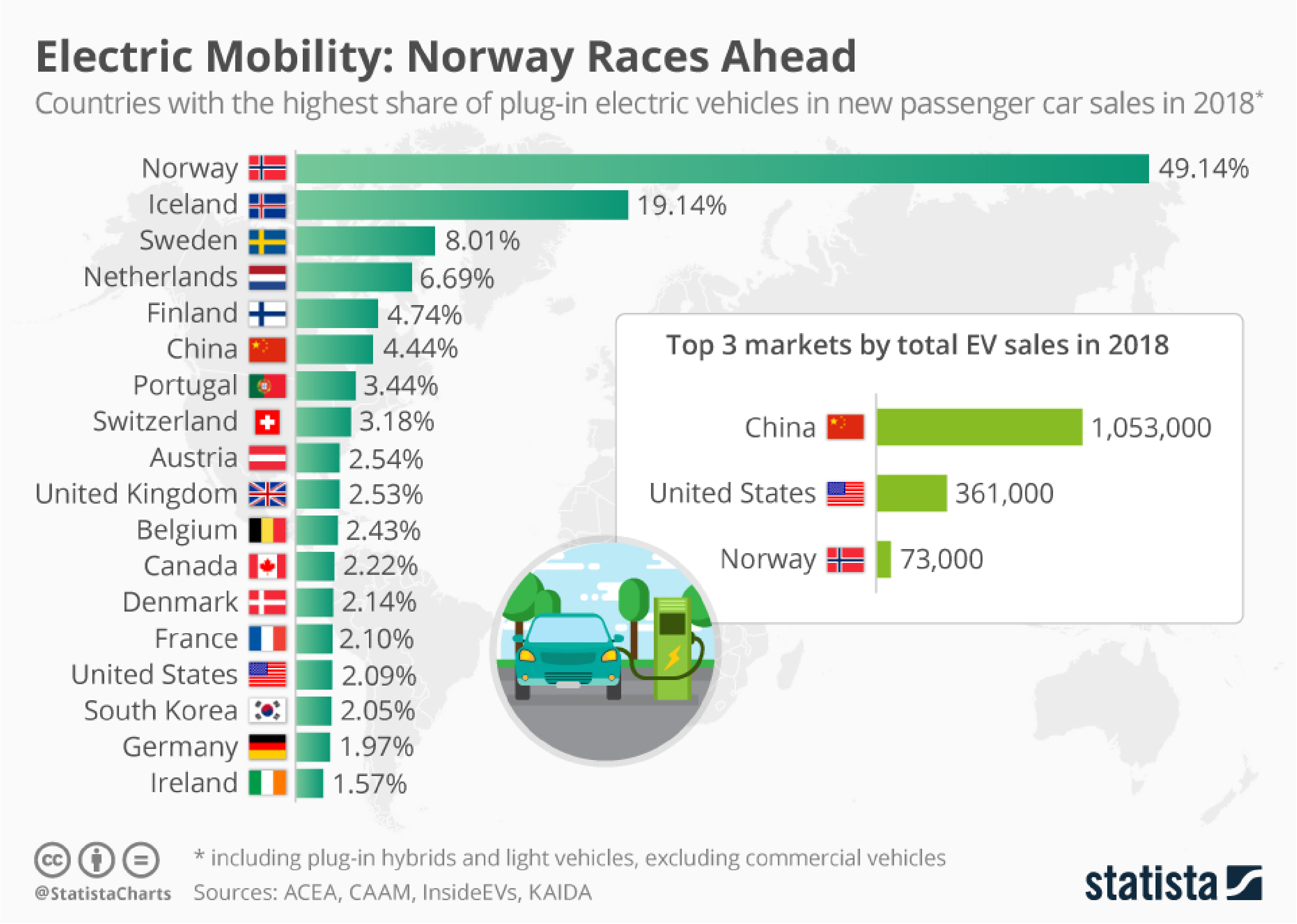
Transportation



Bans of ICE cars

Country	Ban announced	Ban commences
China	2017	no date set
Costa Rica	2018	2021
Denmark	2019	2030
France	2017	2040
Iceland	2018	2030
India	2017	2030
Ireland	2018	2030
Israel	2018	2030
Netherlands	2017	2030
Norway	2017	2025
United Kingdom	2017	2040 – England, Wales, Northern Ireland 2032 – Scotland
Sri Lanka	2017	2040
Sweden	2018	2030

+ many cities..

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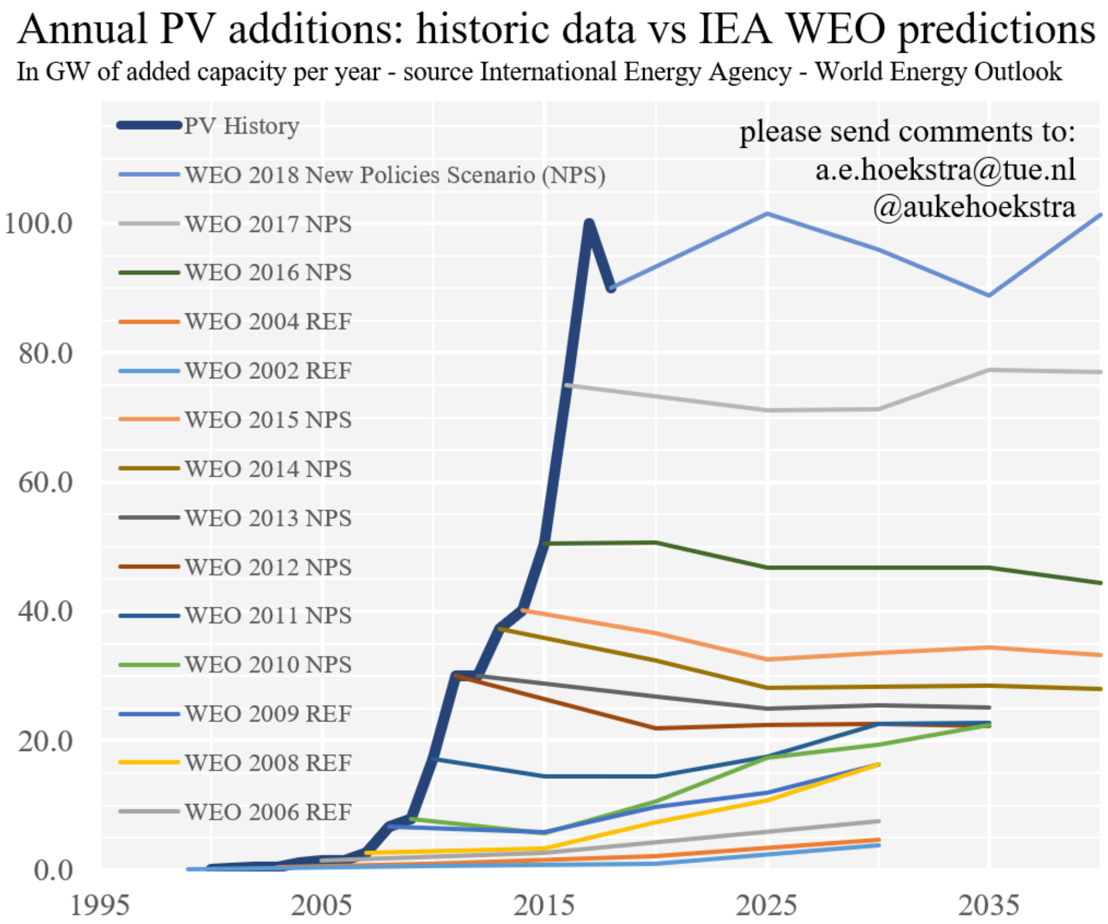
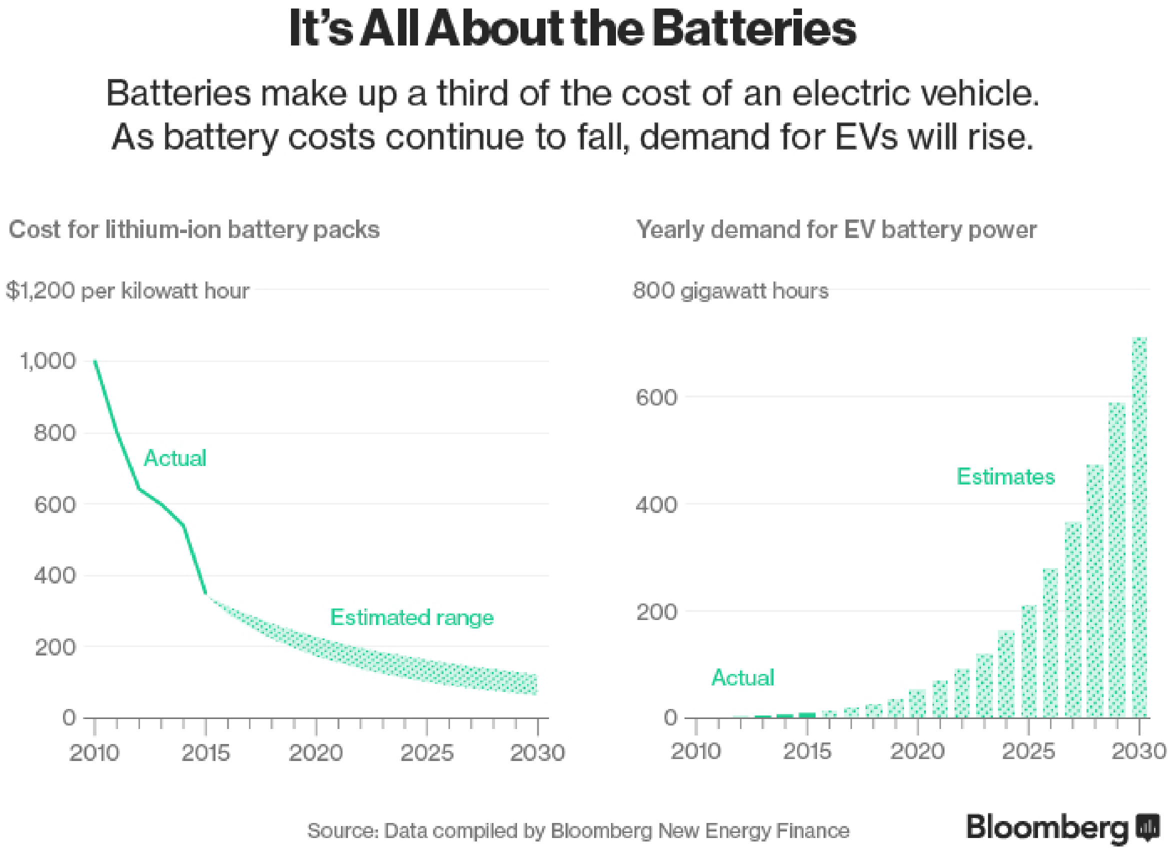
PV and Batteries





Further Related Topics

- Climate change and sustainable development
- Decoupling
- Green growth or degrowth
- Green paradox
- Distributional effects and fairness
- Contagion

Lessons on Decoupling

- Resource scarcity, pollution, and environmental policies are **compatible with sustainable development!**

- **Dynamic** effects of climate policy tend to be ignored, misinterpreted, or underrated -> **Double dividend**

- Important issues make the case for decoupling **stronger**
  - Poor input substitution fosters **sectoral change**
  - Environmental **risks** affect investment and capital accumulation
  - Role of policy for expectations formation
  - Green **expectations** and international **knowledge diffusion** lower the costs of climate policy

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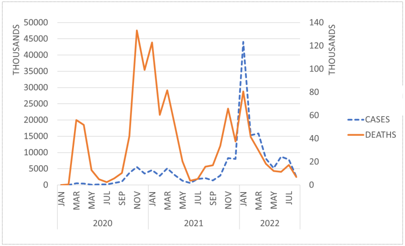
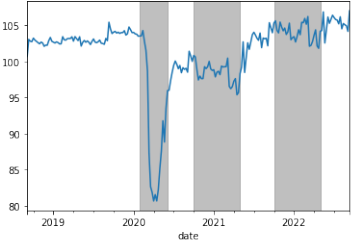
World full of disasters

Misfortunes never come singly

Contagion effect during the COVID-19 pandemic.



COVID cases and deaths in Euro area, January 2020 - July 2022.

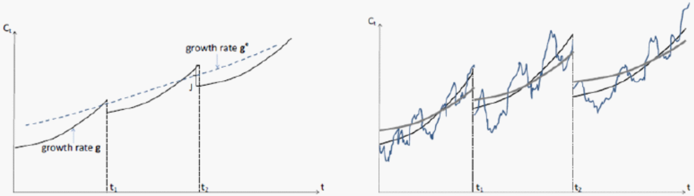


Weekly GDP index of the Euro area.

Secondary disasters bring significant losses

- For earthquakes, Daniell et al. (2017) find that 40 percent of economic losses and deaths result from secondary effects rather than the shaking itself.
- Swiss Re Institute: more than 60 percent of the $76 billion insured natural catastrophe losses in 2018 were due to "secondary peril" events.
- Gallagher Re: economic and insured losses from secondary perils from natural catastrophes are accelerating and surpassing the loss totals from primary perils, leading reinsurers to require higher attachment points.
- Overall economic losses (both insured and non-insured losses) from natural disasters were estimated at US$360 billion in 2022, of which $149 billion (41%) came from primary perils and $211 billion (59%) came from secondary perils.

Recurring Shocks and Growth



(a) Expected (dashed) vs. Poisson-driven (solid) growth.

(b) Poisson and Wiener uncertainty (thin gray) vs. stock effects (bold).

Figure 2: Consumption growth rate (Source: Bretschger and Vinogradova 2017)

In previous approaches shocks may follow the Poisson/Wiener process

- Poisson: discrete *jumps*
- Wiener: continuous *fluctuations* around trend

Main feature: If shocks are driven by a process with independent increments, then abatement is a *constant* share of GDP

Shocks

- Different in size, arrival frequency, (ecological) systems, regions...
- Our general focus
  - Recurring shocks
  - Generic environmental/political/epidemiological/financial context.
  - Varying shock sizes and nonconstant arrival rate
- New feature: *Interlinked shocks*
  - Catastrophic disasters might cause chain reactions and trigger a contagion effect.
  - Public disaster management needs to consider the pattern of occurrence of shocks and interdependencies.
- Particularly relevant for modeling climatic/environmental events

Setup

- Our approach:
  - Analysis of sustainable development and optimum growth in a stochastic endogenous growth model with
    - endogenous investments
    - negative externalities from economic activity
    - contagion among shocks
- Our focus:
  - optimal disaster management
  - optimal growth policies
  - interaction between the two
  - welfare and growth losses from suboptimal policies

Results

- Traditionally, theoretical studies have shown that optimal abatement is a fixed share of GDP.
- Yet, in practice, the situational (reactive) approach is applied
- Examples
  - global
  - local
- Main Contribution: We show that in the presence of interlinked catastrophes, a reactive (stochastic) mitigation is actually optimal.

Modelling disasters: Counting jump processes

- Damages are driven by the counting process denoted by $N_t \in \mathbb{N}_0$ with intensity λ_t:

  $$\mathbb{E}_t\big(N[t, t + \Delta t)\big) = \lambda_t \Delta t,$$

  It can be defined heuristically by:
- Simple counting Poisson process (Martin and Pindyck 2015, Bretschger and Vinogradova 2017)

  $$\lambda_t = \bar{\lambda}$$

  Poisson processes (and more generally Lévy processes) have *independent* and *strictly sationary* increments.
- Poisson process with varying intensity

  $$\lambda_t = \lambda(t)$$

Modelling disasters: Jump processes

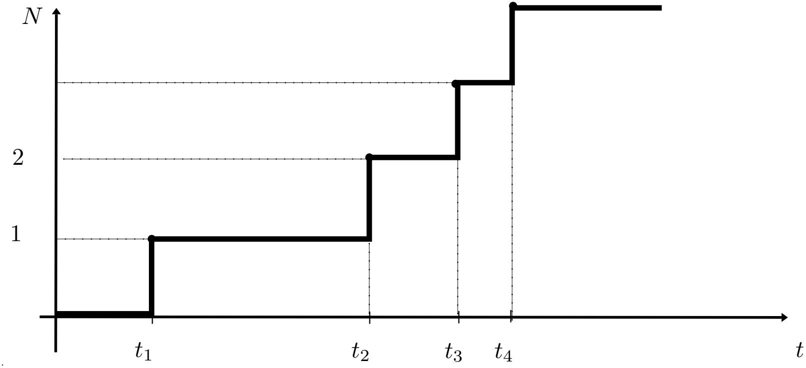


Figure 3: Counting jump process N_t

Hawkes processes

- Hawkes processes generalize Poisson processes by assuming intensity of shocks depends on occurrences of previous shocks

$$\lambda_t = \bar{\lambda} + \sum_{t_j < t} \kappa(t - t_j)$$

- Hawkes process possesses memory and thus allows us to model contagion effects
- Applications include:
  - earthquake modeling (Hawkes 1971, Hawkes 1973)
  - insurance (Hainaut 2016, Stabile and Torrisi 2010 Lesage andothers 2020)
  - finance (see references Hawkes 2018)

Intensity is a stochastic process

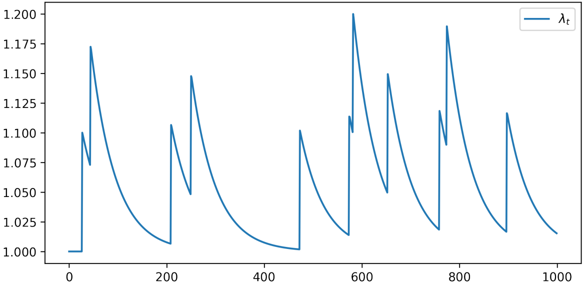


Figure 4: Hawkes process intensity λ_t

- For the exponentially decreasing $\kappa(t) = \alpha \exp\{-\beta t\} 1_{t \geq 0}$, intensity follows

$$d\lambda_t = -\beta(\lambda_t - \bar{\lambda})dt + \alpha dN_t$$

- Hence, two-dimensional process (N_t, λ_t) *is Markovian*

Cluster representation

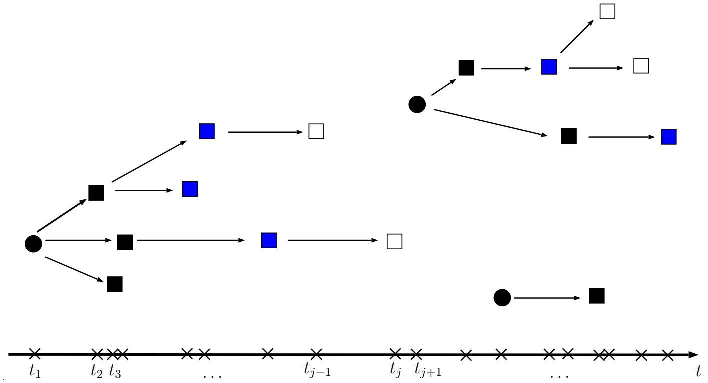


Figure 5: Events' family representation. Circles denote zero-order events while squares of different colors denote descendant events

The branching ratio $\int_0^\infty \alpha e^{-\beta s} ds = \frac{\alpha}{\beta}$ "no-explosion-condition" is assumed to be in $[0, 1)$.

COVID-19 Eurozone 2020–2021 case

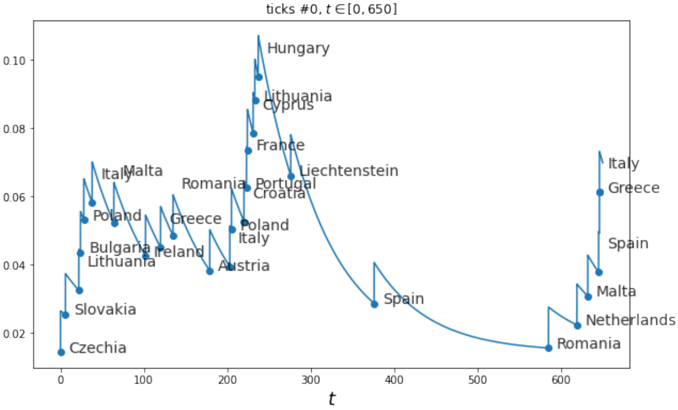


Figure 6: Intensity of the fitted Hawkes process: lockdown spread in Euroarea

Values of the kernel parameters α, β can be estimated from the data.

Assumptions: Economic activity and damages

- Output Y_t is produced with: $Y_t = AK_t$,
- Production generates a negative externality φY_t, which entails damages to capital (disasters), ζ_t, via stochastic arrivals
- Externality can be reduced through mitigation, M_t, by spending a fraction $\theta_t \in [0, \theta_{max}]$ ($\theta_{max} < 1$) of output on "disaster management" : $M_t = \upsilon \theta_t Y_t$
- Net externality is then

$$E_t = \varphi Y_t - \upsilon \theta_t Y_t = (\varphi - \upsilon \theta_t) AK_t$$

- Damages are proportional to the size of the externality with a factor $\gamma \in (0, 1)$:

$$\zeta_t = \gamma E_t = \gamma(\varphi - \upsilon \theta_t) AK_t.$$

Arrivals of interlinked shocks

- We assume Hawkes-driven arrivals:

$$\lambda_t = \bar{\lambda} + \sum_{t_j < t} \kappa(t - t_j)$$

- and exponential decay kernel function in order to keep the process Markovian and be able to use it under the Hamilton-Jacobi-Bellman optimization framework:

$$\kappa(t) = \alpha \exp\{-\beta t\} 1_{t \geq 0},$$

  with $\alpha/\beta < 1$ and $\beta > 0$.

- Other kernels are also possible but they may not allow us to find a closed form solution to the optimization problem.

Preferences

- We assume a logarithmic utility function

$$U(C) = \ln(C)$$

- Another specification depends positively on consumption and negatively on the arrival rate of the disasters:

$$U(C, \lambda) = \ln(C) - \psi(\lambda),$$

Social Planner problem

- The Social Planner maximizes the expected present value of utility over an infinite planning horizon by choosing a consumption path C_t and a disaster-management policy θ_t:

$$\max_{C_t, \theta_t \in [0, \theta_{max}]} \mathbb{E}_0 \left\{ \int_0^\infty U(C_t) e^{-\rho t} \mathrm{d}t \right\},$$

- The Hawkes process N_t (with intensity λ_t) drives arrivals of disasters:

$$\mathrm{d}K_t = [(1 - \theta_t) A K_{t-} - C_t]\mathrm{d}t - \zeta(\theta_t) K_{t-} \mathrm{d}N_t.$$

- Control variables $(C_t, \theta_t)_t$ are assumed to be progressively measurable random variables. They are called *admissible* if capital stock does not vanish and $\theta_t \in [0, \theta_{max}]$.

Solution: Optimal disaster management

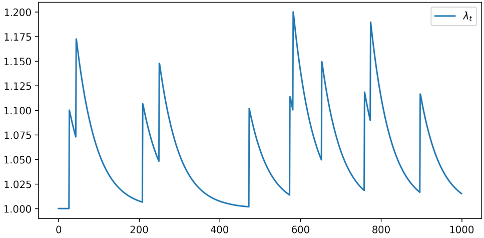
- In general, disaster-management policy may fall into 3 regimes: one interior and two border-control

$$\theta_t^*(\lambda) = \begin{cases} 0, & \text{if} \quad \lambda_t < \lambda^{min} \\ \theta^{max} & \text{if} \quad \lambda_t > \lambda^{max} \\ (0, \theta^{max}) & \quad \text{otherwise,} \end{cases}$$

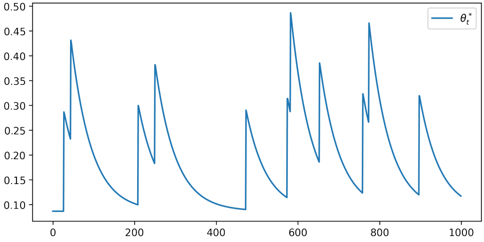
- We assume that $\lambda^{min} = \frac{1}{\upsilon\gamma} - \frac{A\varphi}{\upsilon} > \bar{\lambda}$ and define a truncated process $\tilde{\lambda}_t = \min\{\lambda_t, \lambda^{max}\}$
- Then, the interior solution is:

$$\theta_t^*(\lambda) = \frac{\varphi}{\upsilon} - \frac{1 - \tilde{\lambda}_t \upsilon\gamma}{A\upsilon\gamma}$$

Results: Clustering visualization



Arrival rate of disasters



Mitigation propensity

Solution: Optimal consumption path

- General Keynes-Ramsey rule for optimal consumption:

$$\frac{dC}{C} = \frac{1}{R(C)}\left((1-\theta_t)\,Y_K - \rho + \lambda_t\left[\frac{U_C(\tilde{C})}{U_C(C)}\tilde{K}_K - 1\right]\right)dt + \left[\frac{\tilde{C}}{C} - 1\right]dN_t$$

  where $R(C) \equiv -\frac{CU_{CC}}{U_C}$ is the Arrow-Pratt measure of relative risk aversion

- The red-term corresponds to the standard deterministic Keynes-Ramsey rule: $\frac{dC}{C} = \frac{1}{R(C)}(Y_K - \rho)\,dt$
- Growth rate is stochastic
- For the Logarithmic utility we can completely solve the model, i.e. find value function and policy functions.

Trend growth and expected Growth

- The overall short-run growth rate of consumption is a sum of two components:
  - the "trend growth", i.e. the dt-term in the expression for the Keynes-Ramsey rule:

$$g_t^{tr} = A\left(1 - \frac{\varphi}{\upsilon}\right) - \rho + \frac{1}{\upsilon\gamma} - \tilde{\lambda}_t$$

  - and a jump counterpart given by the dN-term

- The long-term (expected or forecasted) growth rate, denoted by g_τ^e (we set $\tau = 0$):

$$g_0^e = \mathbb{E}_0\left[\frac{\mathrm{d}C_t/\mathrm{d}t}{C_t}\right] = A\left(1 - \frac{\varphi}{\upsilon}\right) + \frac{1}{\upsilon\gamma} - \rho - \mathbb{E}\tilde{\lambda}_t - \mathbb{E}\lambda_t + \upsilon\gamma\mathbb{E}\left[\tilde{\lambda}_t\lambda_t\right].$$

Visualization: Poisson case

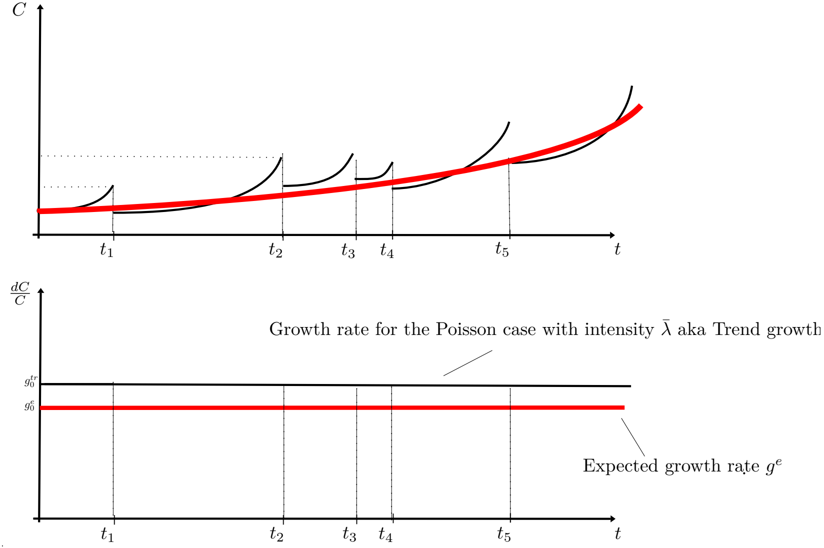


Figure 8: Poisson case: trend and expected consumption growth rates and levels

Visualization: Hawkes case

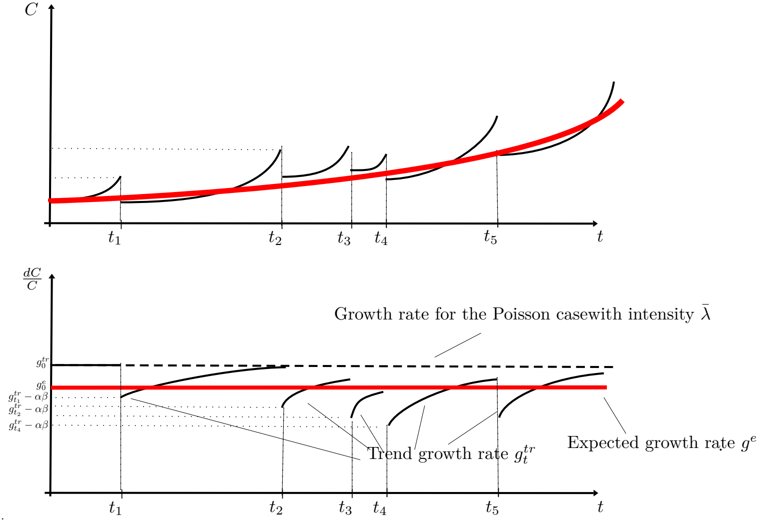


Figure 9: Trend and expected consumption growth rates and levels

Costs of suboptimal policies: Mitigation

- Myopic (Poisson) planner believes that the arrival rate of disasters is constant instead of being stochastic and approximates it with $\lambda^m := \mathbb{E}\lambda_t = \frac{\bar{\lambda}}{1-\alpha/\beta}$.
- Myopic mitigation:

$$\theta^m = \frac{\varphi}{\upsilon} - \frac{1 - \lambda^m \upsilon\gamma}{A\upsilon\gamma}$$

vs

$$\mathbb{E}[\theta_t^*] = \frac{\varphi}{\upsilon} - \frac{1 - \mathbb{E}[\tilde{\lambda}_t]\upsilon\gamma}{A\upsilon\gamma}$$

- Since $\lambda^m > \mathbb{E}[\tilde{\lambda}_t]$, $\theta^m > \mathbb{E}[\theta_t^*]$ for each $t > 0$.
- Myopic planner "overspends" because she misses the right timing.

Costs of suboptimal policies: Growth and welfare

- Myopic short-run growth

$$g^{tr,m} = A\left(1 - \frac{\varphi}{\upsilon}\right) + \frac{1}{\upsilon\gamma} - \rho - \lambda^m$$

vs true

$$g^{tr}_t = A\left(1 - \frac{\varphi}{\upsilon}\right) + \frac{1}{\upsilon\gamma} - \rho - \tilde{\lambda}_t$$

$$\Rightarrow \quad g^{tr} - g^{tr,m} = \lambda^m - \tilde{\lambda}_t$$

- Long-run growth loss:

$$g^{e} - g^{e,m} = \mathbb{E}\,\lambda - \mathbb{E}[\tilde{\lambda}_t] + \upsilon\gamma\Big(\mathbb{E}(\tilde{\lambda}_t\lambda) - (\mathbb{E}\,\lambda)^2\Big)$$

$$\lim_{\lambda^{max}\to+\infty} g^{e} - g^{e,m} = \upsilon\gamma\mathrm{Var}]\lambda] > 0$$

- Corresponding welfare loss: $V - V^m = \frac{1}{\rho}\,\mathbb{E}\,\lambda \ln\left(\frac{\lambda^m}{\lambda}\right)$.

Model Extensions

Model Extensions

- Extension 1: additional source of randomness through fluctuations around the trend (log-normal distribution)

$$dK_t = [(1-\theta_t)AK_{t-} - C_t]dt - \zeta_{t-}dN_t + \varepsilon_{t-}dW_t$$

- Extension 2: additional source of randomness through damage size (any distribution)

$$dK_t = [(1-\theta_t)AK_{t-} - C_t]dt - Z_t\zeta_{t-}dN_t \quad (+\varepsilon_{t-}dW_t)$$

- Extension 3: Hawkes-driven damages. Functions $\delta = \delta(\lambda)$ and $\gamma = \gamma(\lambda)$ are twice continuously differentiable and satisfy the quadratic growth constraint

$$dK_t = [(1-\theta_t)AK_{t-} - C_t]dt - Z_t\gamma(\lambda)E_{t-}dN_t + \delta(\lambda)E_{t-}dW_t$$

$\rightarrow$ stochastic volatility model.

Model Extensions

- The extended model writes:

$$\max_{(C_t,\theta_t)} \quad \mathbb{E}\int_0^{+\infty} u(C_t)e^{-\rho t}dt$$

$$\text{s.t. } dK_t = [(1-\theta_t)AK_{t-} - C_t]dt - \gamma Z_t E_{t-} dN_t + \delta E_{t-} dW_t$$

$$d\lambda_t = \beta[\bar{\lambda} - \lambda_t]dt + \alpha dN_t$$

- The size of the damages from disasters is proportional to the size of the externality and is scaled up by the random component Z_t.
- Z_t is a bounded non-negative continuous random variable independent of the Hawkes process and of the Brownian motion.

Extended model solution

- The standard HJB equation reads:

$$\rho V(K_t, \lambda_t) = \max_{C_t, \theta_t} \{u(C_t) + \frac{1}{\mathrm{d}t}\mathbb{E}_t \mathrm{d} V(K_t, \lambda_t)\}, \tag{1}$$

- The choice of the utility function guides us to consider a candidate solution of the HJB equation (1) in the form:

$$V(K, \lambda) = \rho^{-1} \ln(K) + g(\lambda)$$

- At time $t \geq 0$ the optimal consumption $C_t^* = \rho K_t$, and the optimal mitigation policy is $\theta^* = \theta_t^*(\lambda_t)$:

$$\theta^*(\lambda) = \mathrm{argmax}_{\theta \in [0, \theta_{max}]} R(\theta, \lambda) \tag{2}$$

$$R(\theta, \lambda) \equiv (1-\theta)A - \frac{1}{2}\delta^2\Gamma^2 + \lambda \int \ln\Big(z(1-\omega)\Big)\mathrm{d}\nu(z).$$

where $\Gamma = (\varphi - \theta v)A$ and $\omega = \gamma Z \Gamma$.

- One can show, that $g(\lambda)$ is a differentiable function, so that a classic solution exists.

Extension 1: Wiener uncertainty

$$\max \int_0^{+\infty} u(C_t, \lambda_t) e^{-\rho t} dt$$

$$dK = [(1-\theta)AK - C]dt - \gamma E_t dN_t + \delta E_t dW_t$$

The HJB:

$$\rho V(K, \lambda) = \max\{U(c, \lambda) + \frac{1}{dt} \mathbb{E}_t \, dV\}$$

where

$$dV = V_k[(1-\theta)Y - C]dt - \beta\lambda V'_\lambda dt + \frac{1}{2}\upsilon^2 V''_{kk} dt + [\tilde{V} - V]dN_t$$

$\tilde{V} = V(\tilde{K}, \tilde{\lambda})$, where $\tilde{K} = (1-\omega)K$ and $\tilde{\lambda} = \lambda + \alpha dN_t$.

...Wiener uncertainty

FOCs:

$$\begin{cases} U_C = V_k \\ -V_k' Y + \frac{1}{2} V_{kk}''(\upsilon^2)_\theta + \lambda_t \mathbb{E}_t \tilde{V}_{\tilde{k}}' \tilde{K}_\theta = 0. \end{cases}$$

- If the Hawkes uncertainty dominates, i.e. $\delta \to 0$, then

$$\theta = \theta^H + \upsilon\lambda\Gamma^H\delta^2 + o(\delta^2),$$

where $\Gamma^H = \Gamma_t^H = (1 - \lambda_t\gamma\upsilon)/\gamma$ and $\theta^H = \theta_t^H = \frac{\varphi}{\upsilon} - \frac{1-\lambda_t\upsilon\gamma}{A\upsilon\gamma}$ are the random processes for the optimal Γ and θ in the case of pure Hawkes uncertainty obtained earlier.

- If the Wiener uncertainty dominates , i.e. $\gamma \to 0$, then

$$\theta = \theta^W + \frac{\lambda}{\delta\upsilon A}\gamma + o(\gamma)$$

where $\theta^W = \frac{\varphi}{\upsilon} - \frac{1}{(A\upsilon\delta)^2}$ is mitigation share in the pure Wiener case.

Model extensions: Random Jump size

$$\max \int_0^{+\infty} u(C_t, \lambda_t) e^{-\rho t} dt$$

$$dK = [(1-\theta)AK - C]dt - \gamma Z E dN_t,$$

where Z is a positive bounded random variable independent of the Hawkes process. FOCs:

$$\begin{cases} U_C = V_k \\ -V_k' Y + \lambda_t \int \tilde{V}_{\tilde{k}}' \tilde{K}_\theta \mathrm{d}\nu(z) = 0. \end{cases}$$

Let Z have a Bernoulli distribution with outcomes b and s representing big and small relative loss ($b > s > 0$) taking place with probabilities $(p, 1-p)$. The second condition yields

$$\lambda_t \gamma v \left[\frac{b}{1-b\omega} p + \frac{s}{1-s\omega}(1-p) \right] - 1 = 0.$$

This is again a quadratic equation in $\omega \equiv \gamma\Gamma = \gamma(\varphi - v\theta)A$ (or θ).

Model extensions: Random Jump size

- If $s = 1$, i.e. "small" events are like before, then $\theta > \theta^*$.

- If we assume that the probability p of a big disaster is small, we can derive the following asymptotic expansion

$$\theta = \theta_Z + \Delta_1 p + o(p).$$

where $\theta_Z = \frac{\varphi}{\upsilon} - \frac{1-\lambda_t \upsilon \gamma s}{A \upsilon \gamma s}$ and Δ_1 stands for $\frac{\lambda(b-s)}{|A(s-b(1-\lambda\upsilon\gamma s))|}$. If $\lambda\upsilon\gamma s$ is small then $\Delta_1 \approx \frac{\lambda}{A}$.

$$\theta_Z \gtrless \theta_H \quad \Leftrightarrow \quad s \lessgtr 1$$

Further Model Extensions

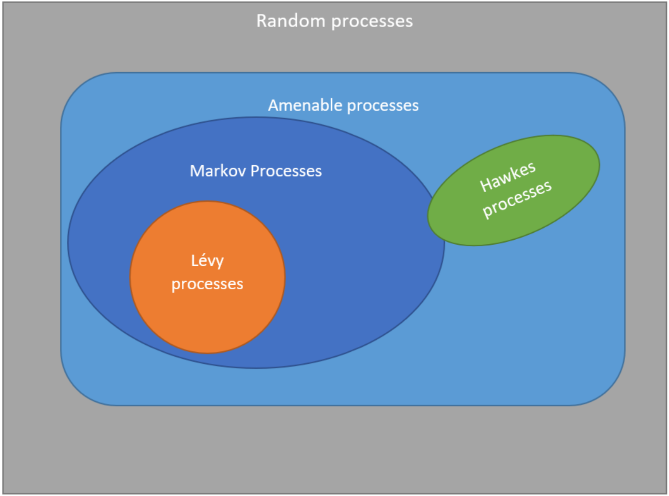
- results beyond the logarithmic-utility
- different policies, e.g., a policy that could affect parameters of the Hawkes kernel density itself
- stronger link to externality (e.g. $\lambda(E_t)$)
- derive a multidimensional analogue of the model shocks, where processes of different nature would be mutually exciting
- calibrate the model and provide quantitative results: welfare cost of uncertainty, optimal mitigation propensity, growth costs of contagion

Conclusion

- We developed a general-equilibrium endogenous growth model with stochastic environmental shocks stemming from economic activity.
  - Negative externalities of economic activity affect the size of primary shocks that are associated with subsequent disasters.
  - To study the nature of interrelated shocks, the model uses the concept of Hawkes process, which is a novelty in this field.
  - We derive closed-form solutions of the economic growth rate, the optimal spending on disaster prevention (abatement).
- *Main finding:* The optimal disaster-mitigation policy is stochastic (reactive),
- Approximation of Hawkes-driven disaster arrivals by Poisson arrivals leads to growth and welfare losses
- Additional small-scale fluctuations increase optimal mitigation propensity
- Random damage size, in general, has an ambiguous effect on optimal mitigation

APPENDIX

Random processes



Random processes